EE 435

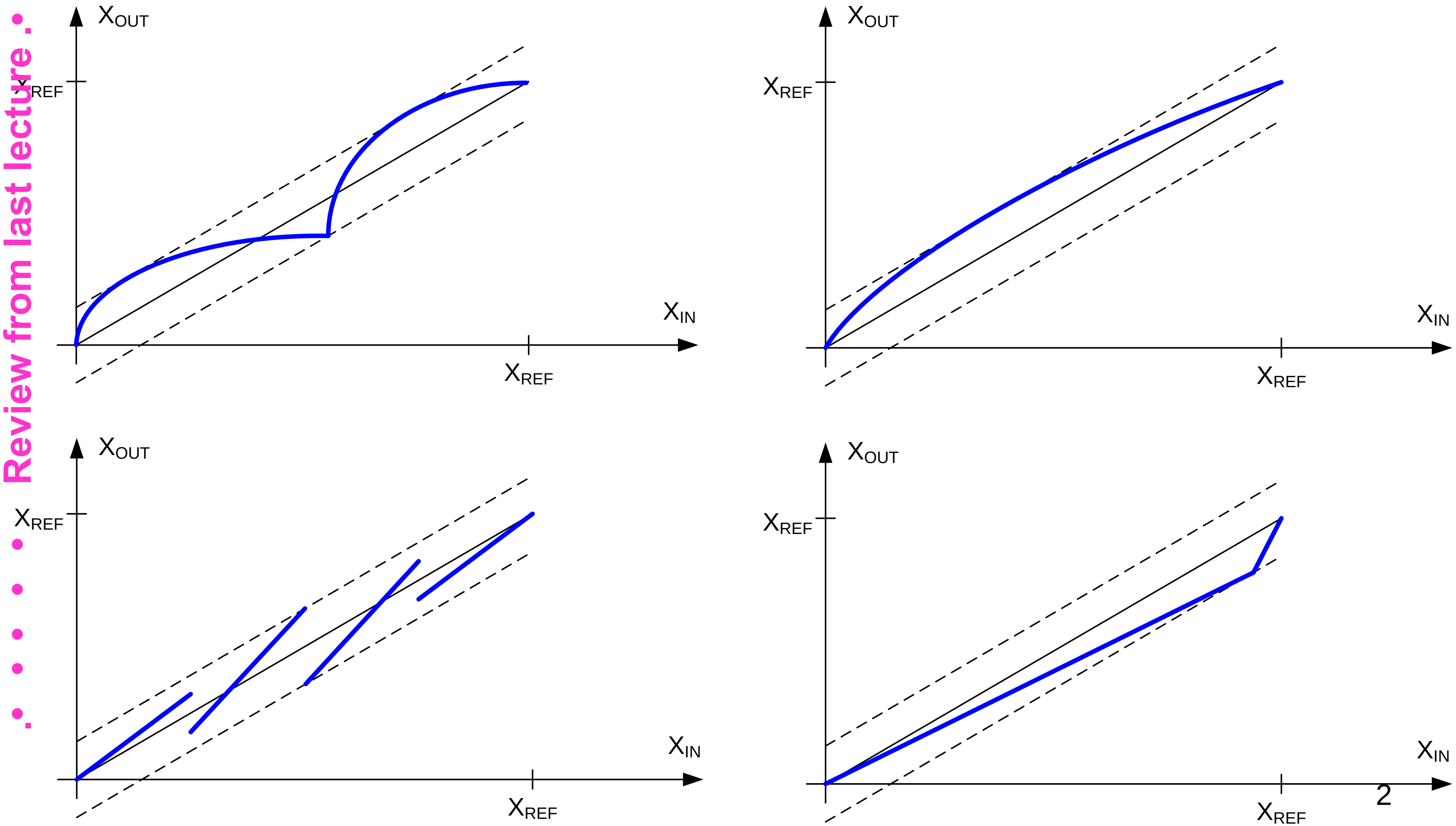
Lecture 29

Data Converters

- Spectral Performance
    - Windowing
- Quantization Noise

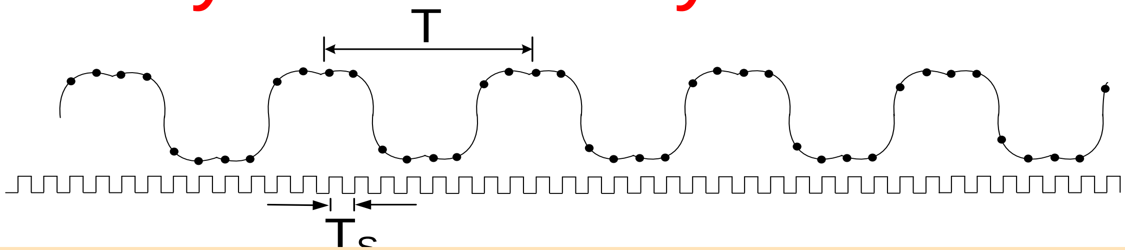
INL Often Not a Good Measure of Linearity

Four identical INL with dramatically different linearity



Review from last lecture

Why is this a Key Theorem?



Review from last lecture

THEOREM: Consider a periodic signal with period T=1/f and sampling period T_S=1/f_S. If N_P is an integer and x(t) is band limited to f_{MAX}, then

$$\left|A_m\right| = \frac{2}{N}\left|X\left(mN_P+1\right)\right| \qquad 0 \le m \le h-1$$

and $X(k)=0$ for all k not defined above

where $\left\langle X(k)\right\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\left\langle x(kT_S)\right\rangle_{k=0}^{N-1}$

N=number of samples, N_P is the number of periods, and h = Int$\left(\frac{f_{MAX}}{f} - \frac{1}{N_P}\right)$

- DFT requires dramatically less computation time than the integrals for obtaining Fourier Series coefficients
- Can easily determine the sampling rate (often termed the Nyquist rate) to satisfy the band limited part of the theorem

Distortion Analysis

How are spectral components determined?

By integral

$$A_k = \frac{1}{\omega T}\left(\int_{t_1}^{t_1+T} f(t)e^{-jk\omega t}dt + \int_{t_1}^{t_1+T} f(t)e^{jk\omega t}dt\right)$$

or

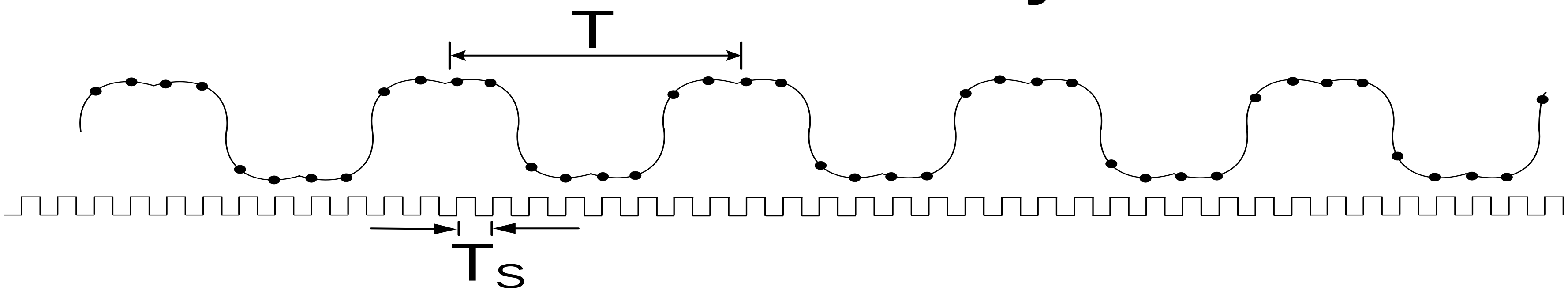
$$a_k = \frac{2}{\omega T}\int_{t_1}^{t_1+T} f(t)\sin(kt\omega)dt \qquad b_k = \frac{2}{\omega T}\int_{t_1}^{t_1+T} f(t)\cos(kt\omega)dt$$

Integral is very time consuming, particularly if large number of components are required

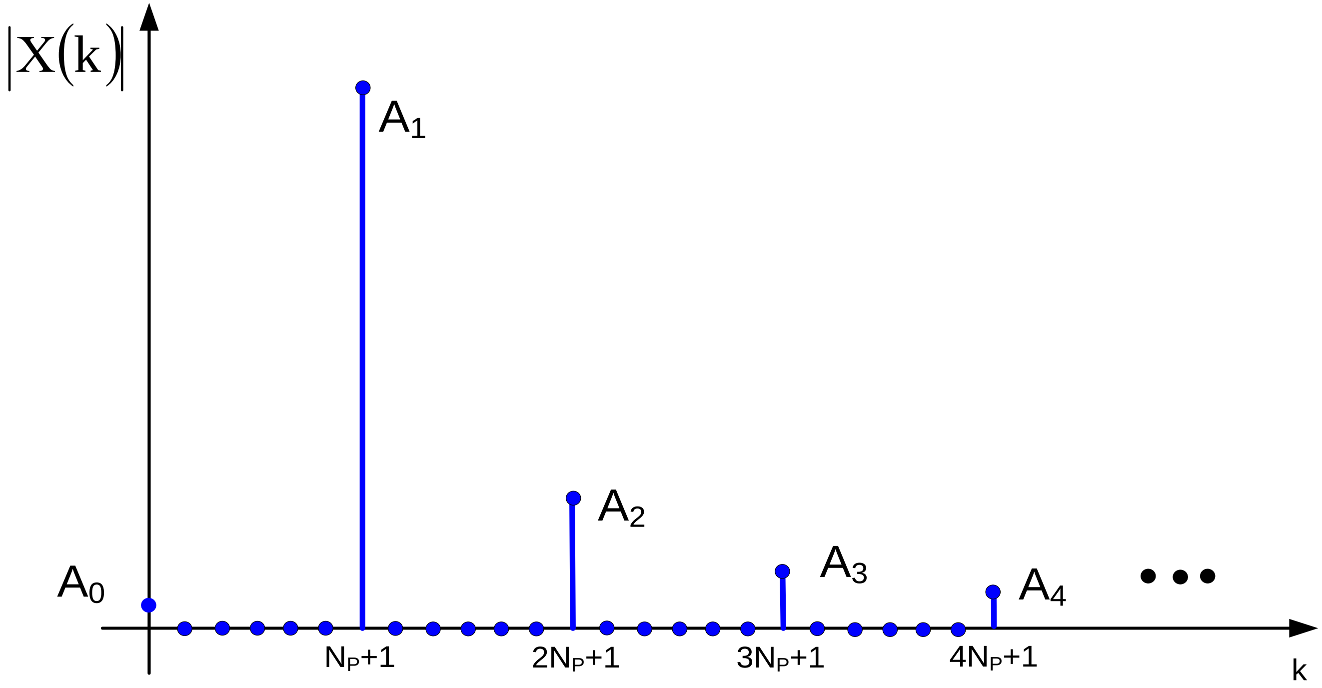
By DFT (with some restrictions that will be discussed)

By FFT (special computational method for obtaining DFT)

Distortion Analysis



If the hypothesis of the theorem are satisfied, we thus have



Review from last lecture

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
- Windowing

Considerations for Spectral Characterization

DFT Length and NP

- DFT Length and NP do not affect the computational noise floor
- Although not shown here yet, DFT length does reduce the quantization noise floor coefficients but not total quantization noise

If we assume E_{QUANT} is fixed and no signal present

$$E_{QUANT} \cong \sqrt{\sum_{k=1}^{2^{n_{DFT}}} A_k^2}$$

(these are now the DFT coefficients due to quantization noise, not computation noise)

If the A_k's are constant and equal

$$E_{QUANT} \cong A_k 2^{n_{DFT}/2}$$

Solving for A_k, obtain

$$A_k \cong \frac{E_{QUANT}}{2^{n_{DFT}/2}}$$

If input is full-scale sinusoid with only amplitude quantization with n-bit res,

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

(this expression is actually independent of input waveform)

Considerations for Spectral Characterization

DFT Length

$$E_{QUANT} \cong \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1}}$$

Substituting for E_{QUANT}, obtain

$$A_k \cong \frac{X_{REF}}{\sqrt{3} \bullet 2^{n+1} 2^{n_{DFT}/2}}$$

This value for A_k thus decreases with the length of the DFT sampline window

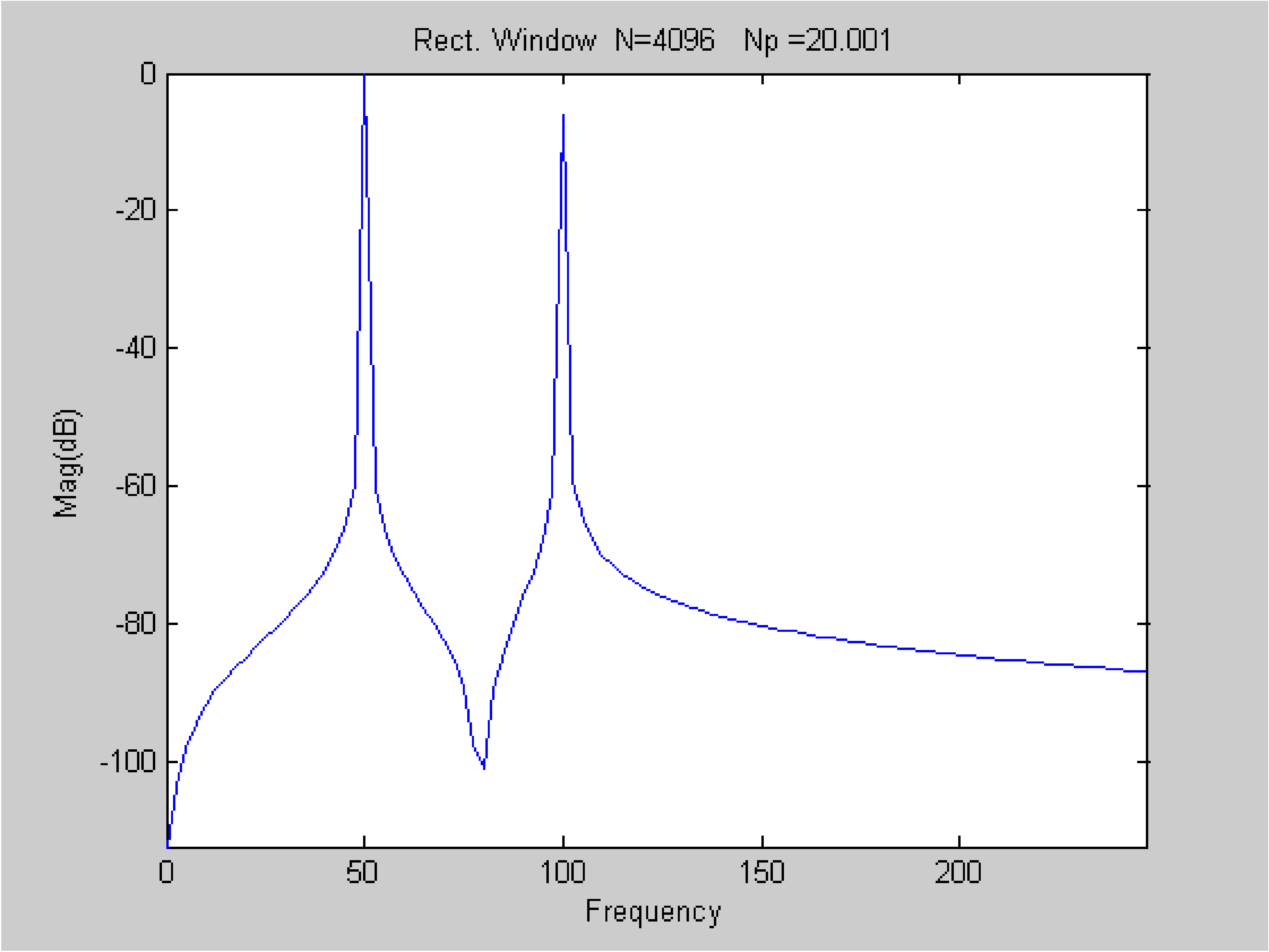
Example: if n=16, n_{DFT}=12 (4096 pt transform), and X_{REF}=1V,
then A_k=6.9E-8V (-143dB),

(Note A_k>> computational noise floor (-310dB for Matlab) for all practical n, n_{DFT})

Considerations for Spectral Characterization

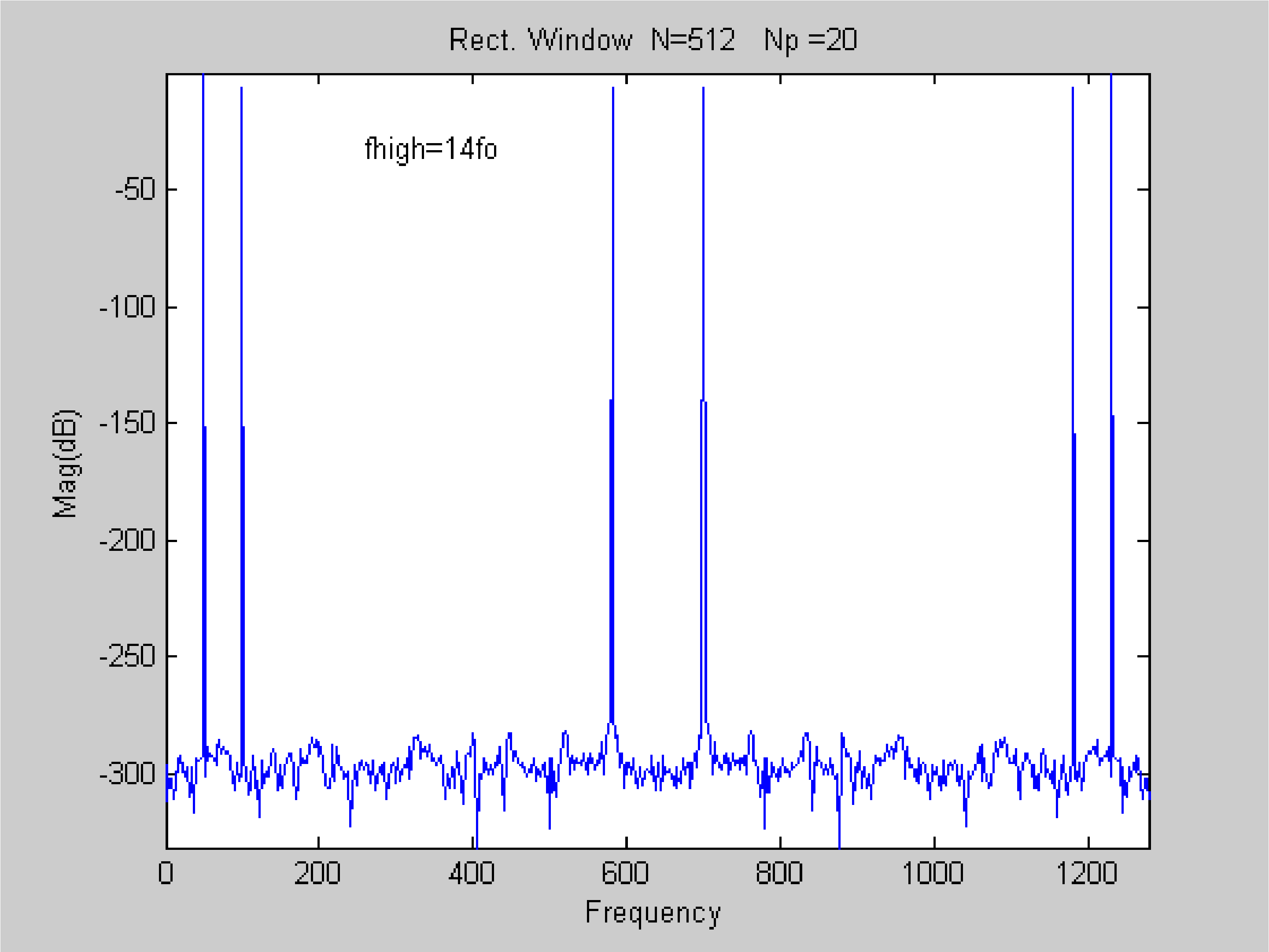
- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
- Windowing

Spectral Response with Non-coherent Sampling

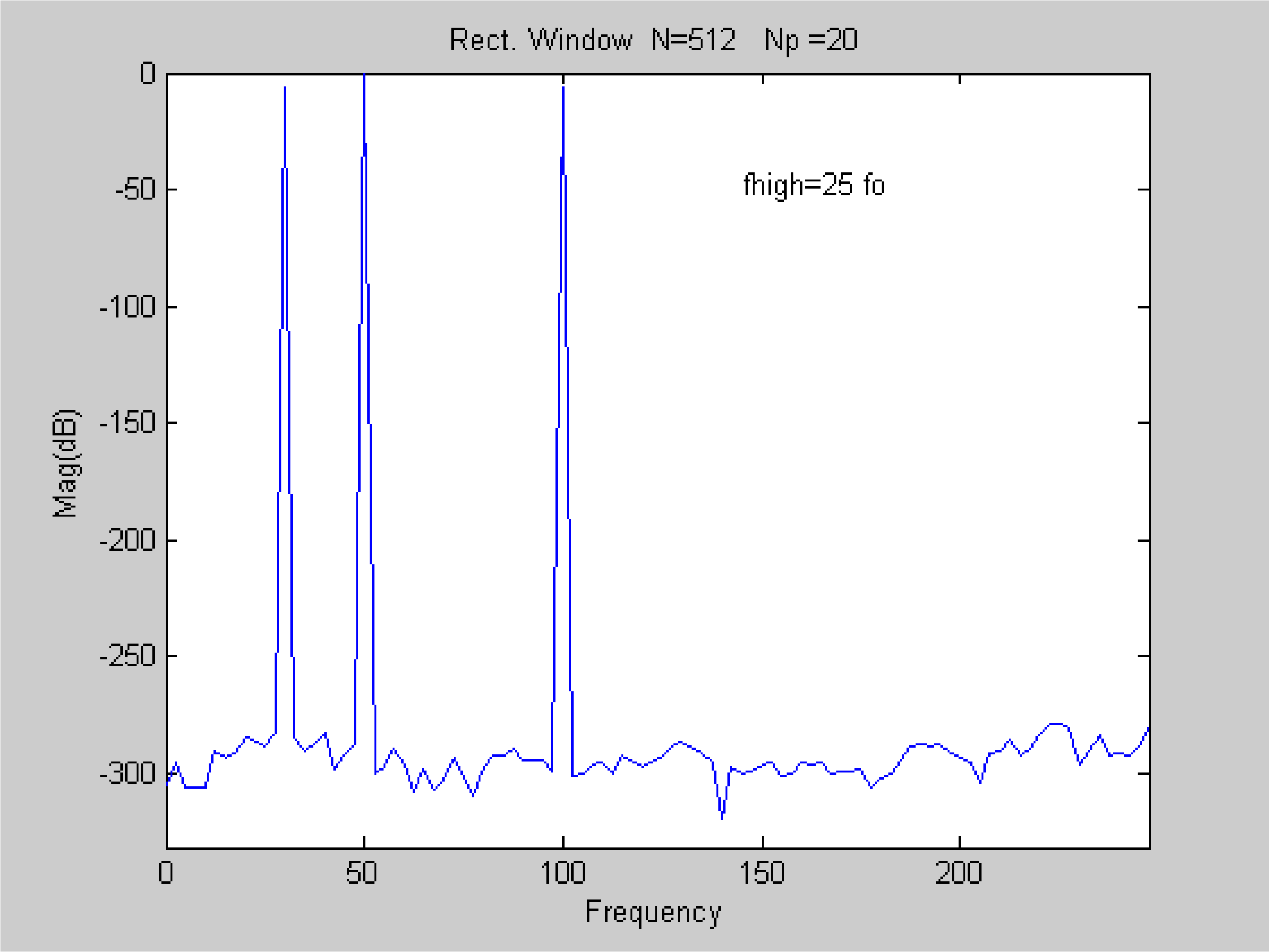


(zoomed in around fundamental)

Effects of High-Frequency Spectral Components



Effects of High-Frequency Spectral Components



(zoomed in around fundamental)

Observations

- Aliasing will occur if the band-limited part of the hypothesis for using the DFT is not satisfied
- Modest aliasing will cause high frequency components that may or may not appear at a harmonic frequency
- More egregious aliasing can introduce components near or on top of fundamental and lower-order harmonics
- Important to avoid aliasing if the DFT is used for spectral characterization

Considerations for Spectral Characterization

- Tool Validation
- DFT Length and NP
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing

Are there any strategies to address the problem of requiring precisely an integral number of periods to use the FFT?

Windowing is sometimes used

Windowing is sometimes misused

Windowing

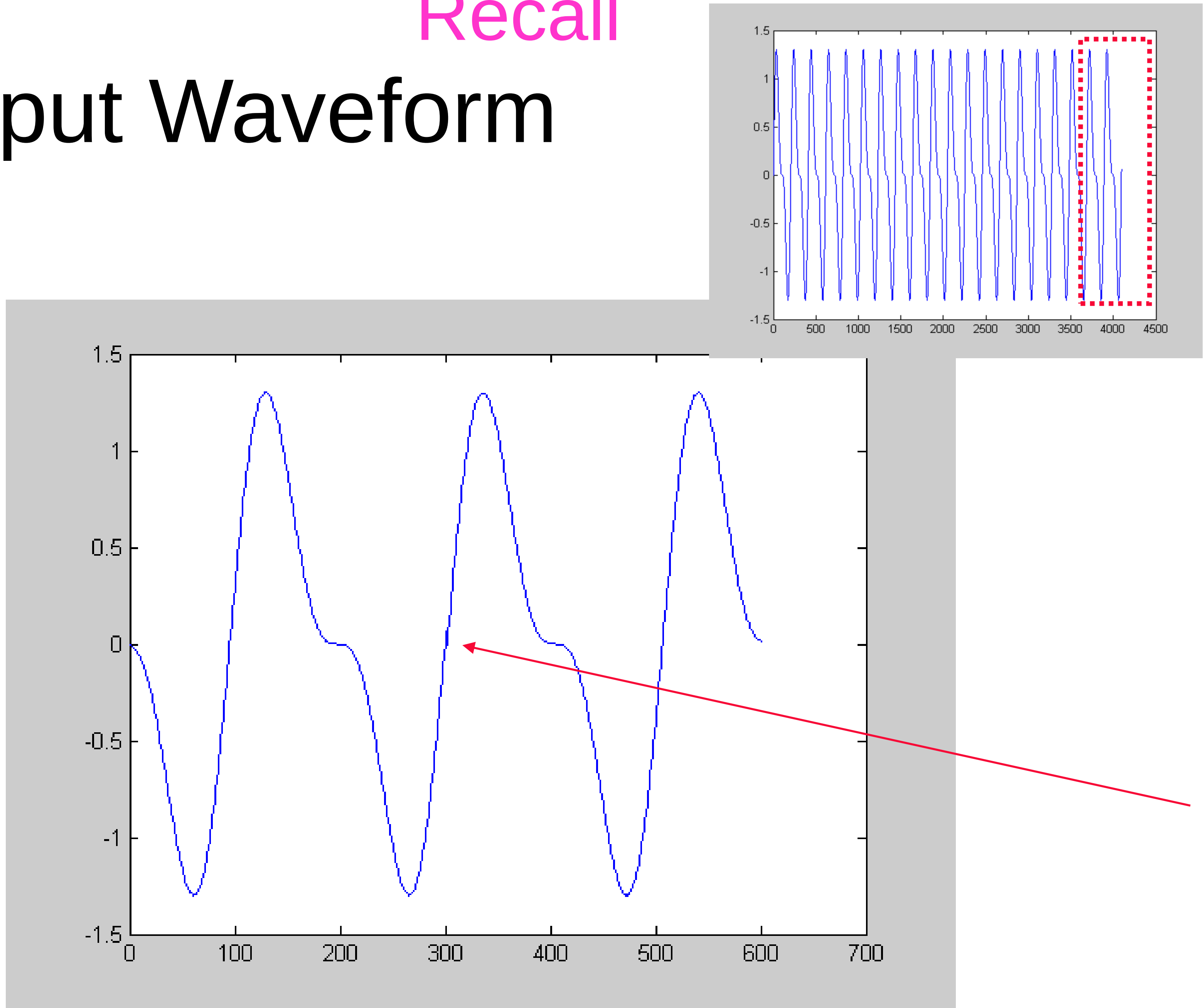
Windowing is the weighting of the time domain function to maintain continuity at the end points of the sample window

Well-studied window functions:

- Rectangular (also with appended zeros)
- Triangular
- Hamming
- Hanning
- Blackman

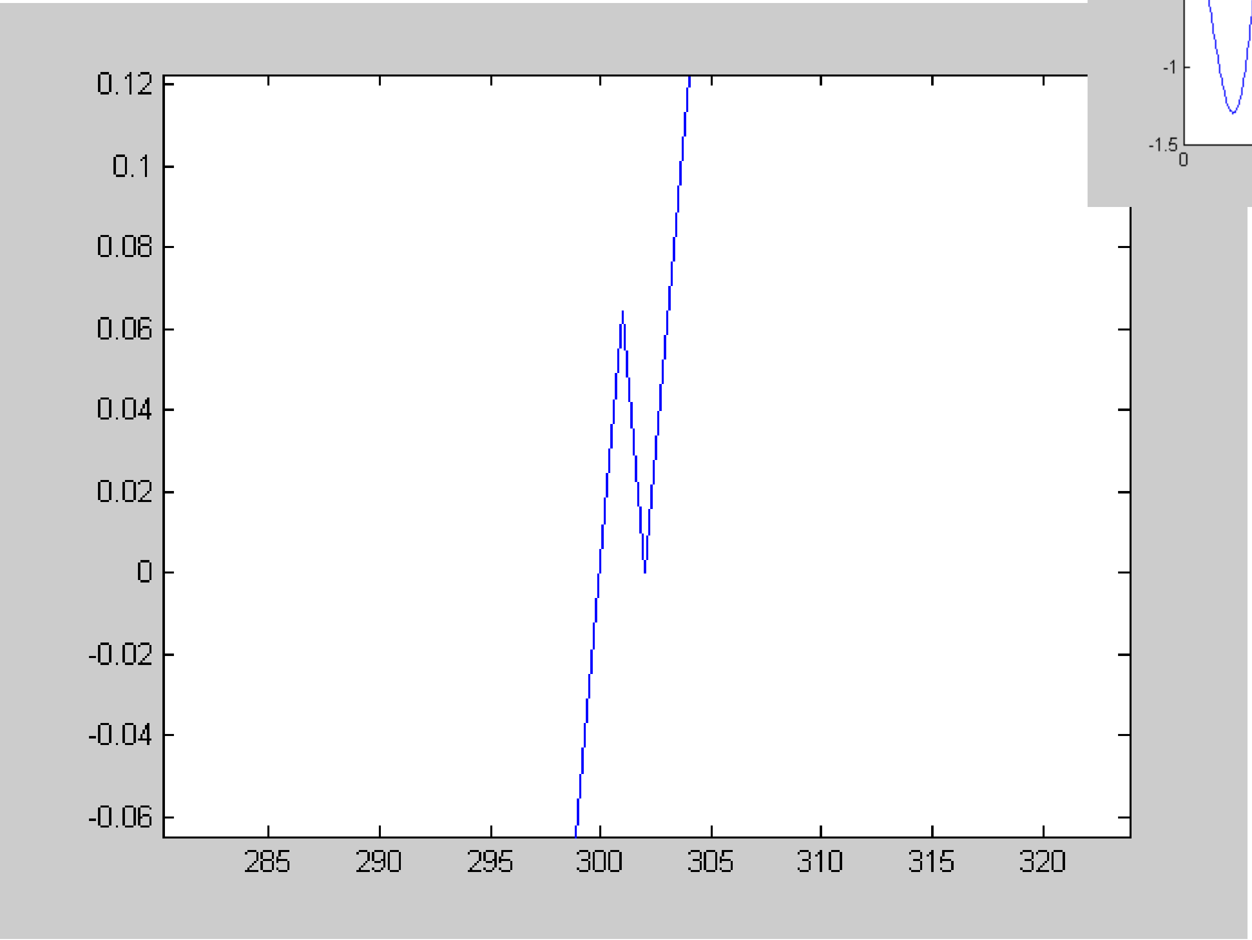
Recall

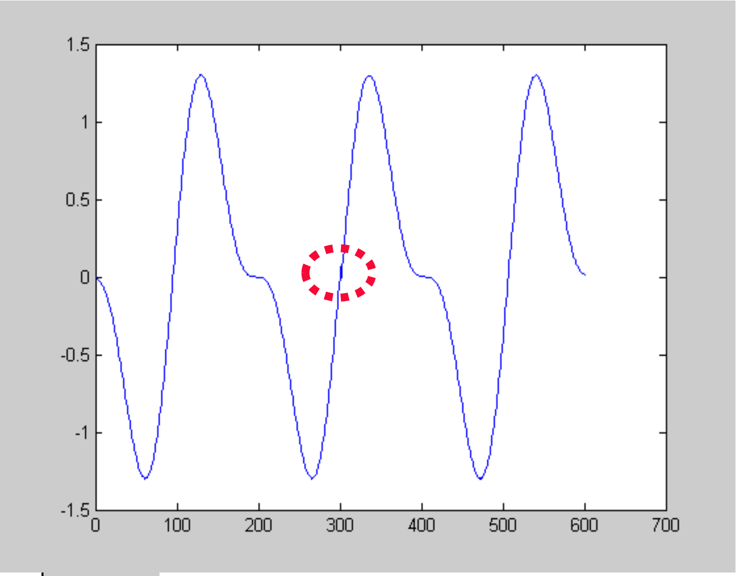
Input Waveform



Recall

Input Waveform





Rectangular Window

Sometimes termed a boxcar window

Uniform weight

Can append zeros

Without appending zeros equivalent to no window

Rectangular Window

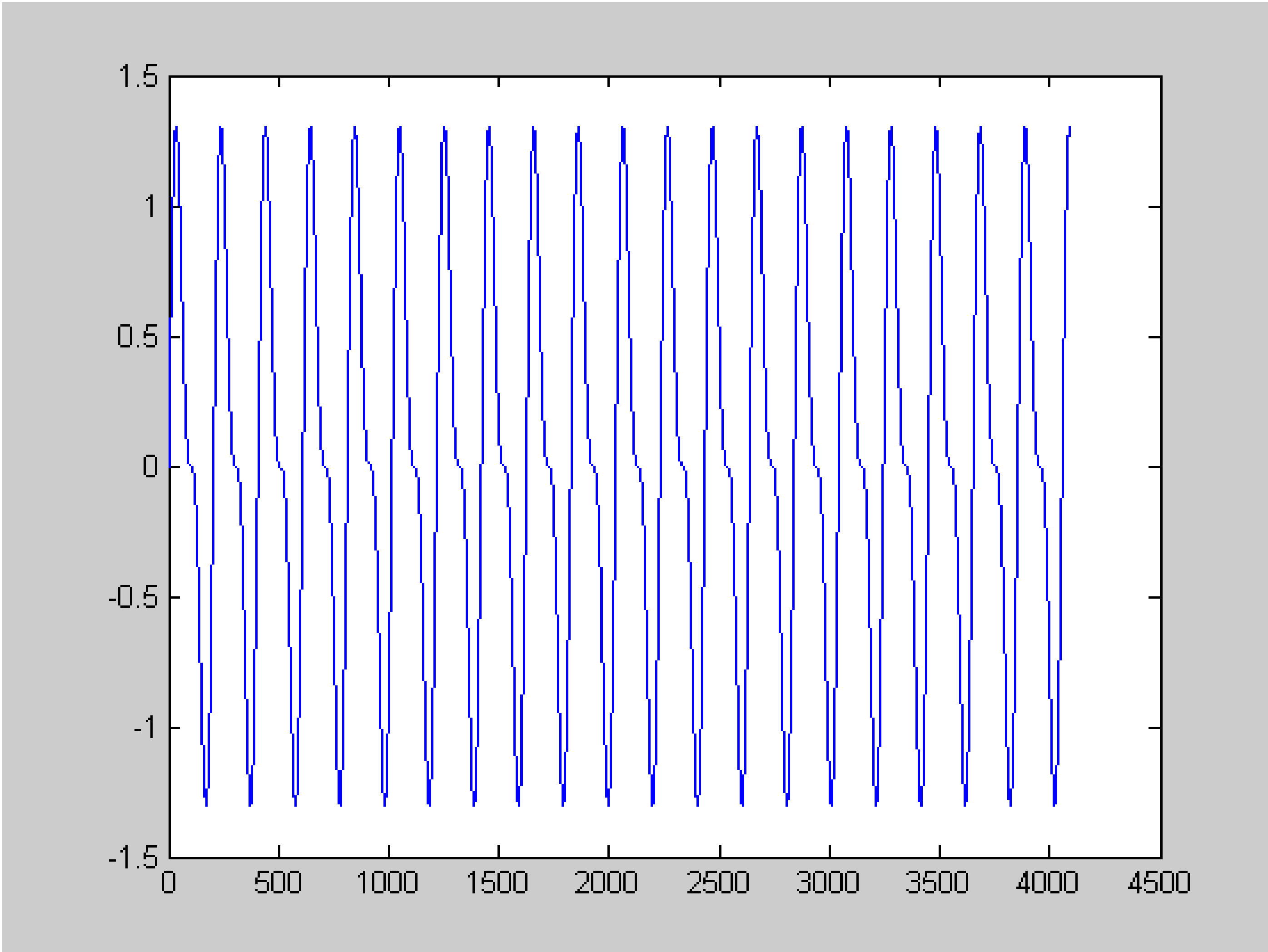
Assume f_{SIG}=50Hz

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

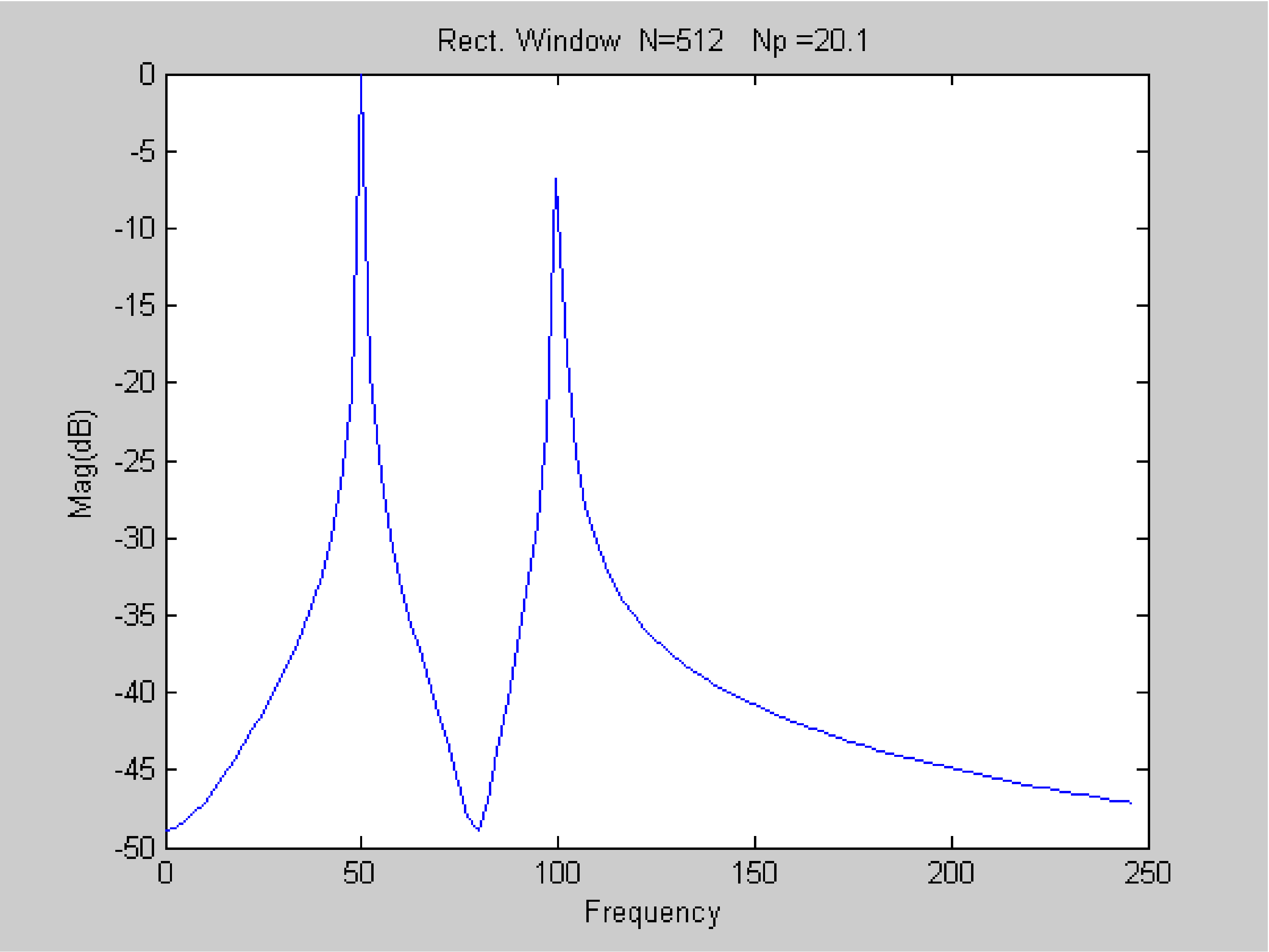
$$\omega = 2\pi f_{SIG}$$

Consider N_P=20.1 N=512

Rectangular Window

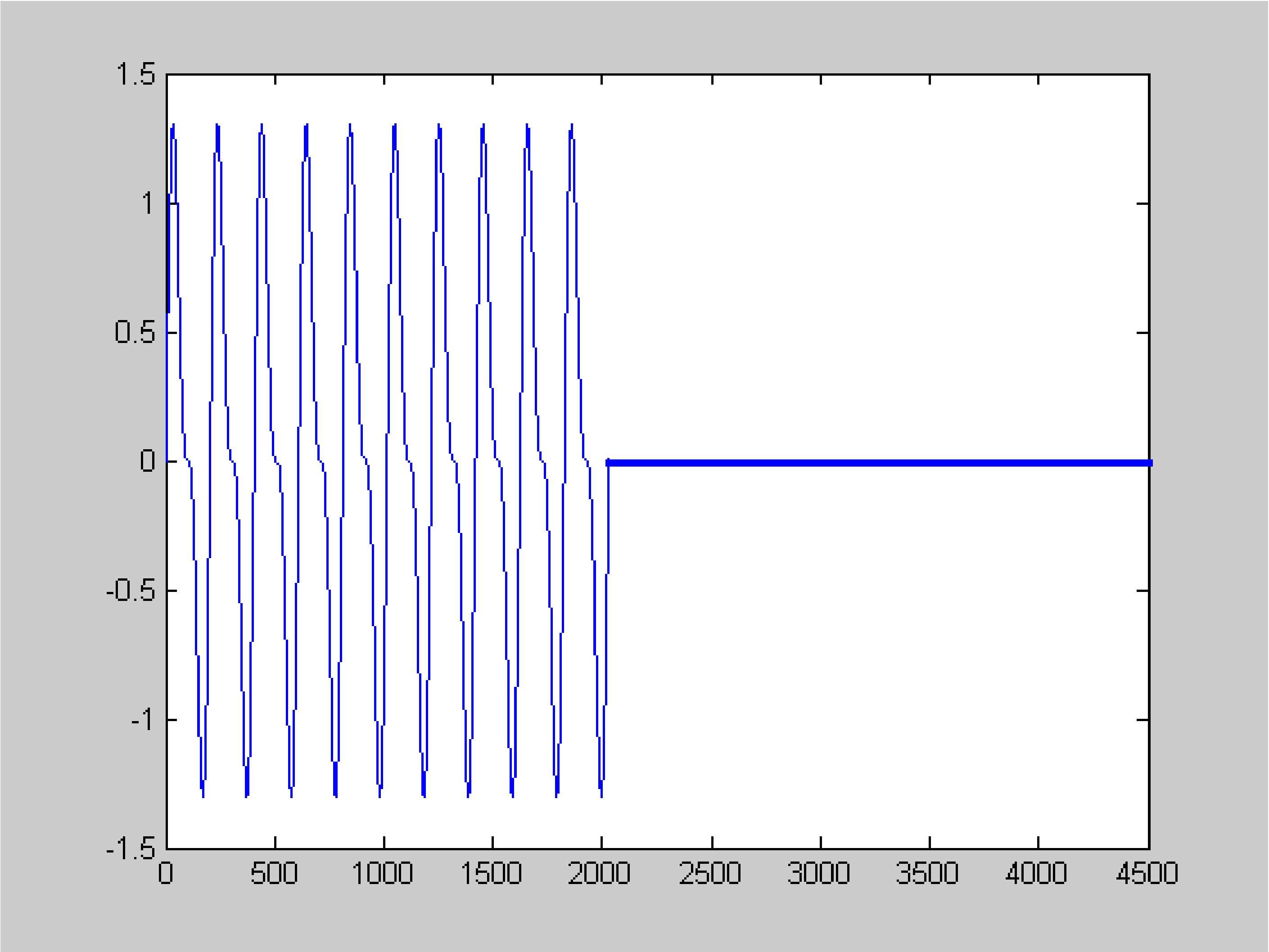


Spectral Response with Non-coherent sampling



(zoomed in around fundamental)

Rectangular Window (with appended zeros)



Rectangular Window

Columns 1 through 7

-48.8444 -48.7188 -48.3569 -47.7963 -47.0835 -46.2613 -45.3620

Columns 8 through 14

-44.4065 -43.4052 -42.3602 -41.2670 -40.1146 -38.8851 -37.5520

Columns 15 through 21

-36.0756 -34.3940 -32.4043 -29.9158 -26.5087 -20.9064 -0.1352

Columns 22 through 28

-19.3242 -25.9731 -29.8688 -32.7423 -35.1205 -37.2500 -39.2831

Columns 29 through 35

-41.3375 -43.5152 -45.8626 -48.0945 -48.8606 -46.9417 -43.7344

Rectangular Window

Columns 1 through 7

-48.8444 -48.7188 -48.3569 -47.7963 -47.0835 -46.2613 -45.3620

Columns 8 through 14

-44.4065 -43.4052 -42.3602 -41.2670 -40.1146 -38.8851 -37.5520

Columns 15 through 21

-36.0756 -34.3940 -32.4043 -29.9158 -26.5087 -20.9064 -0.1352

Columns 22 through 28

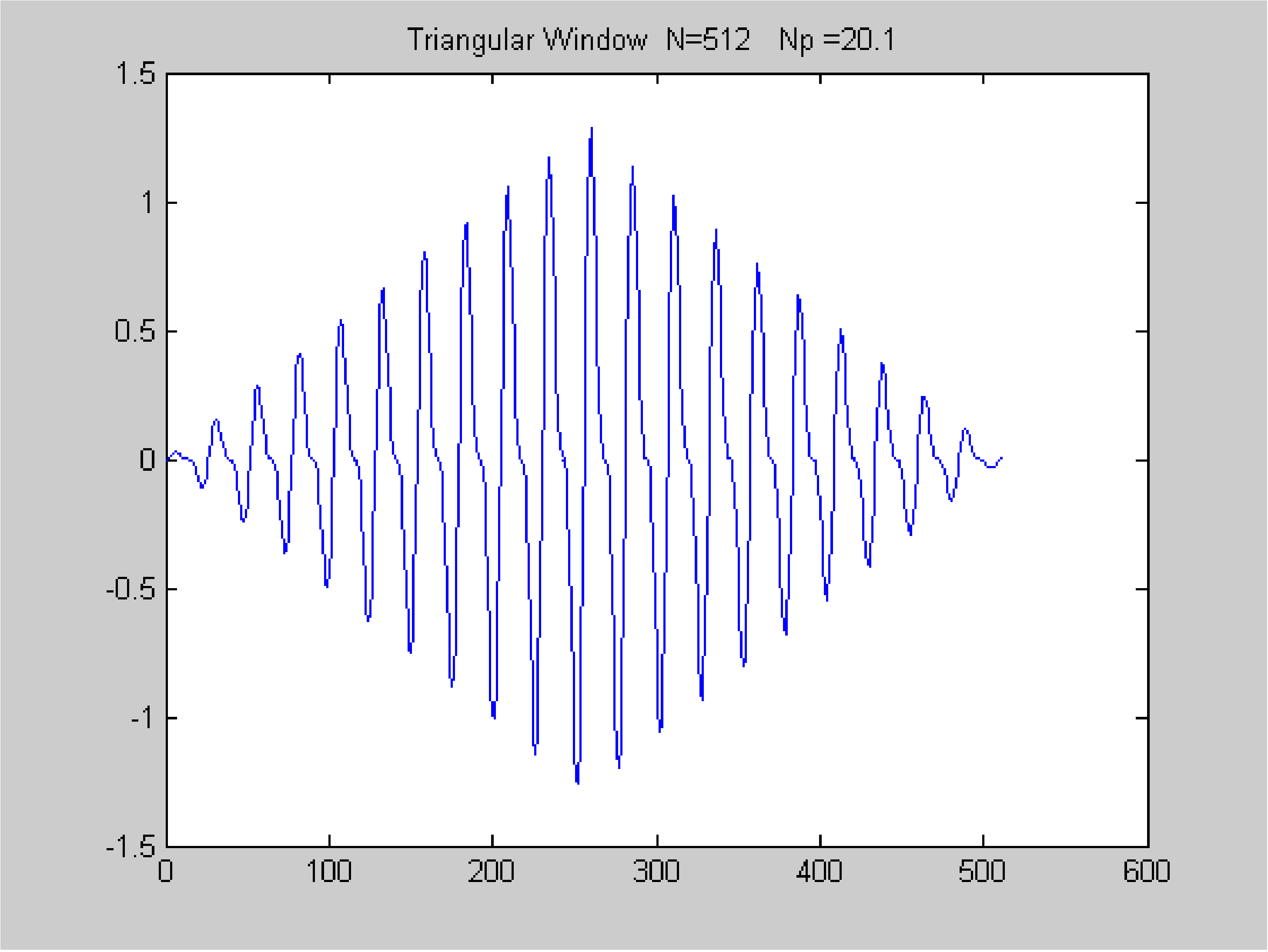
-19.3242 -25.9731 -29.8688 -32.7423 -35.1205 -37.2500 -39.2831

Columns 29 through 35

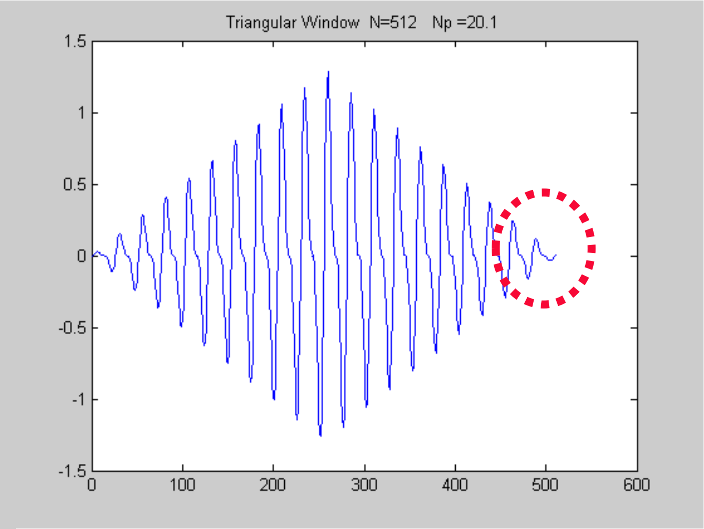
-41.3375 -43.5152 -45.8626 -48.0945 -48.8606 -46.9417 -43.7344

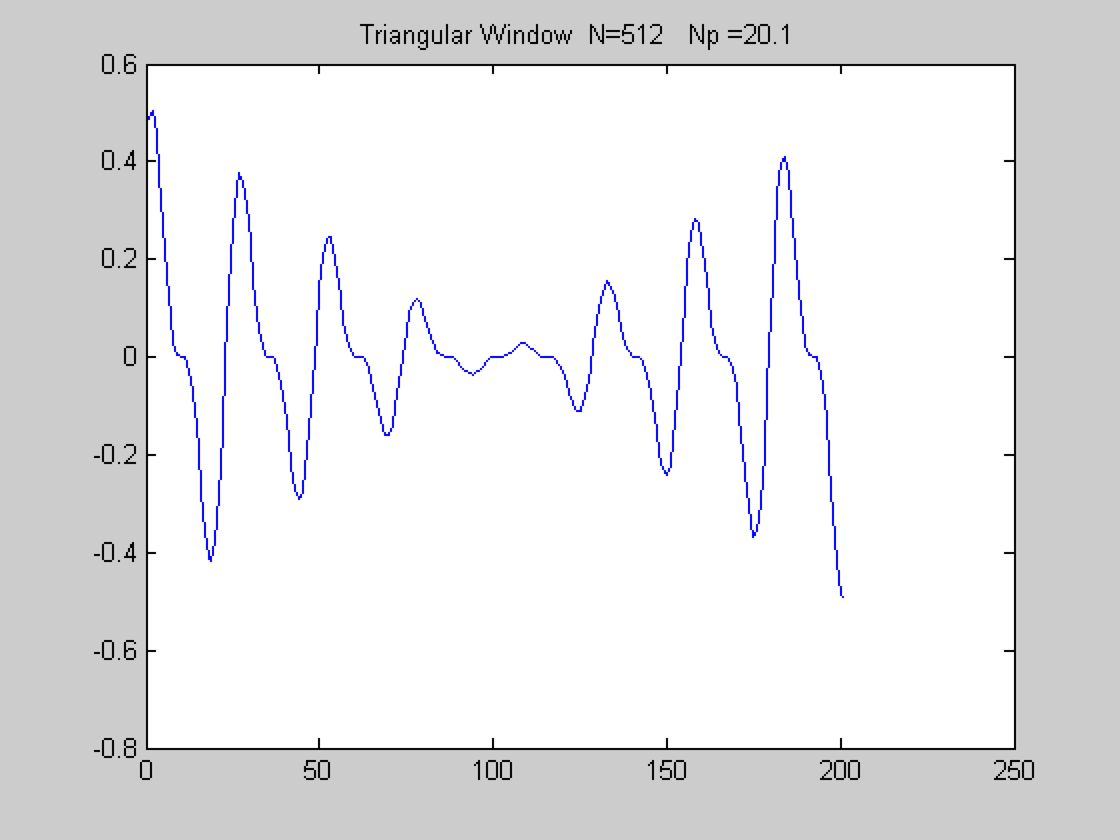
Energy spread over several frequency components

Triangular Window

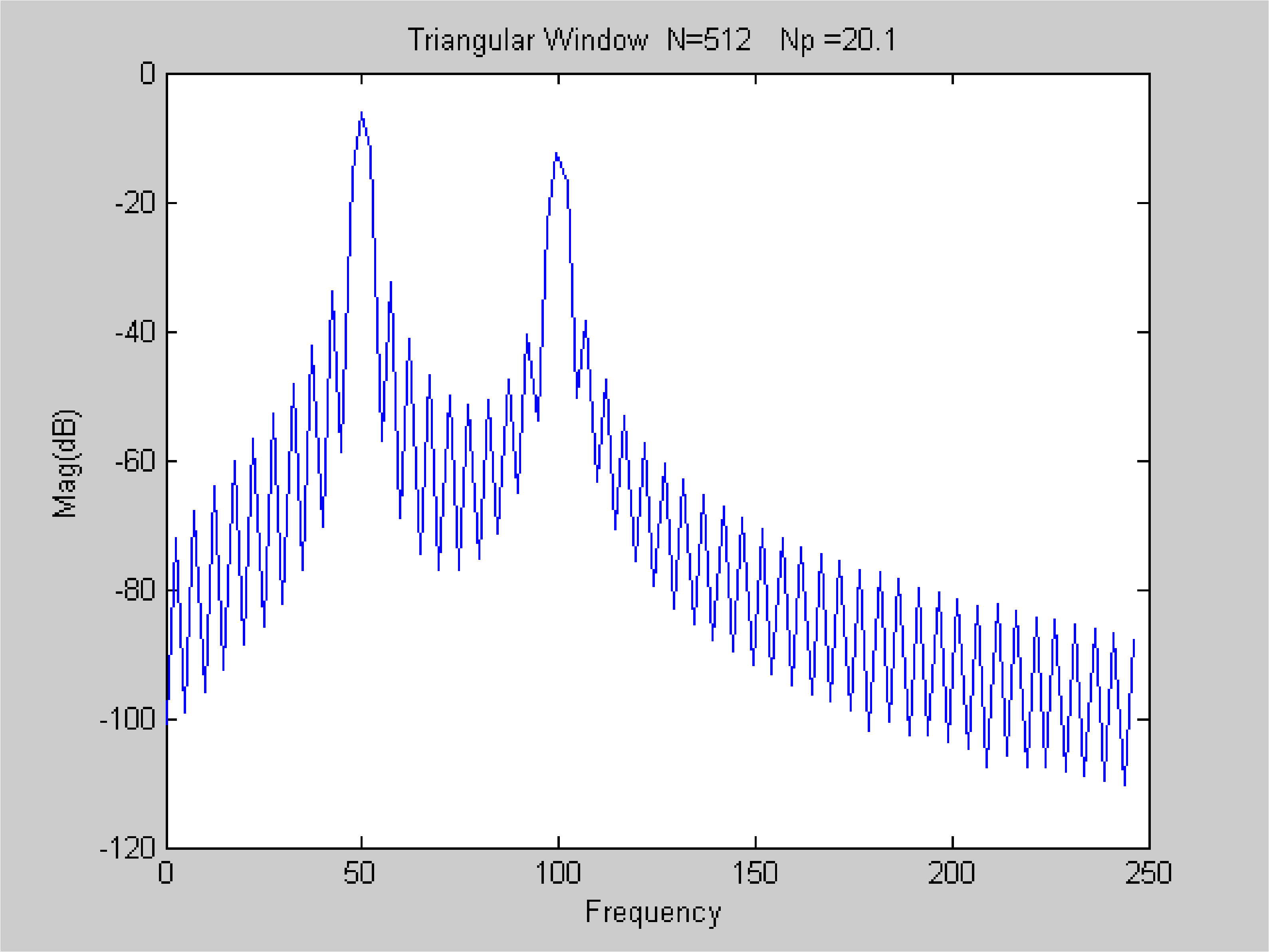


Triangular Window



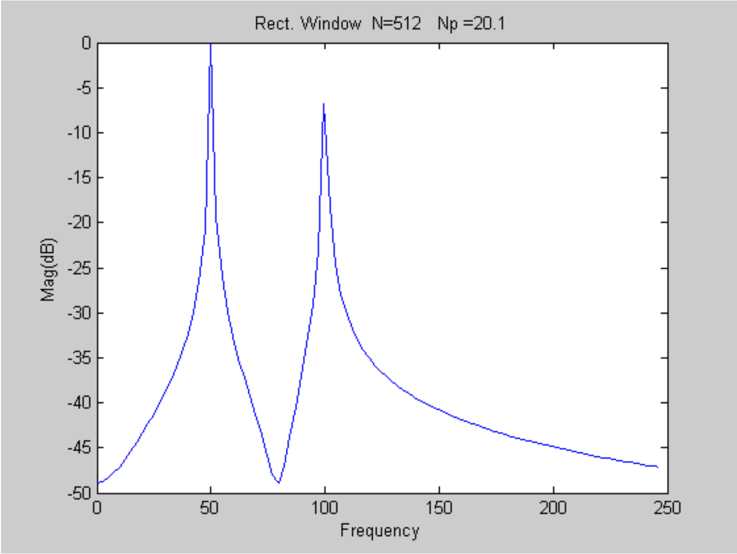


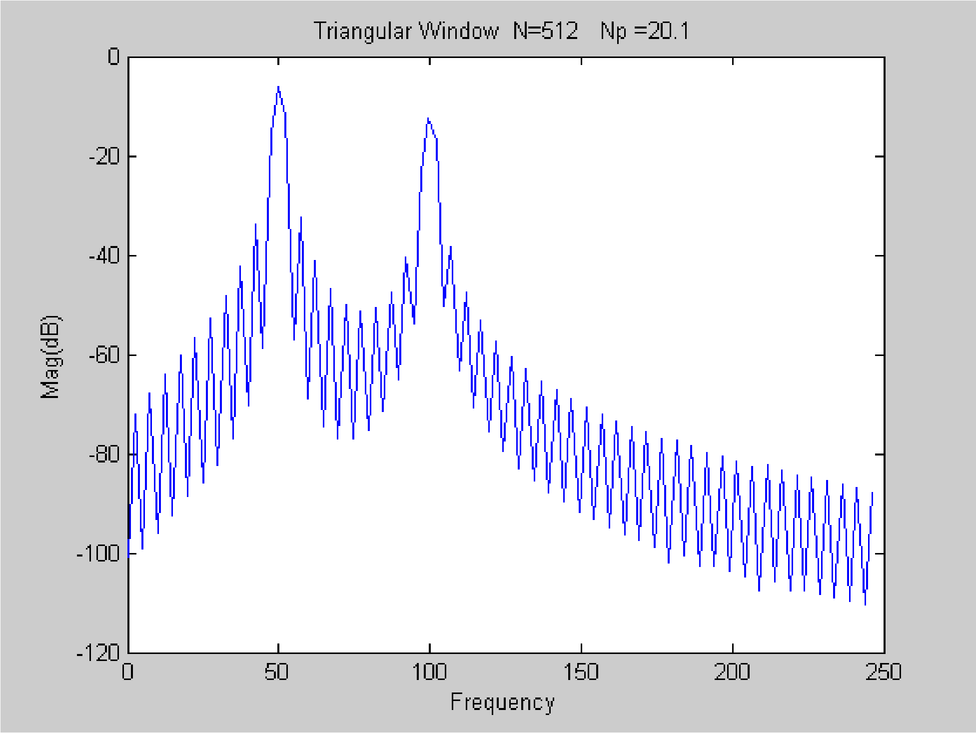
Spectral Response with Non-Coherent Sampling and Windowing



(zoomed in around fundamental)

Triangular Window





Triangular Window

Columns 1 through 7

-100.8530 -72.0528 -99.1401 -68.0110 -95.8741 -63.9944 -92.5170

Columns 8 through 14

-60.3216 -88.7000 -56.7717 -85.8679 -52.8256 -82.1689 -48.3134

Columns 15 through 21

-77.0594 -42.4247 -70.3128 -33.7318 -58.8762 -15.7333 -6.0918

Colu

-12.

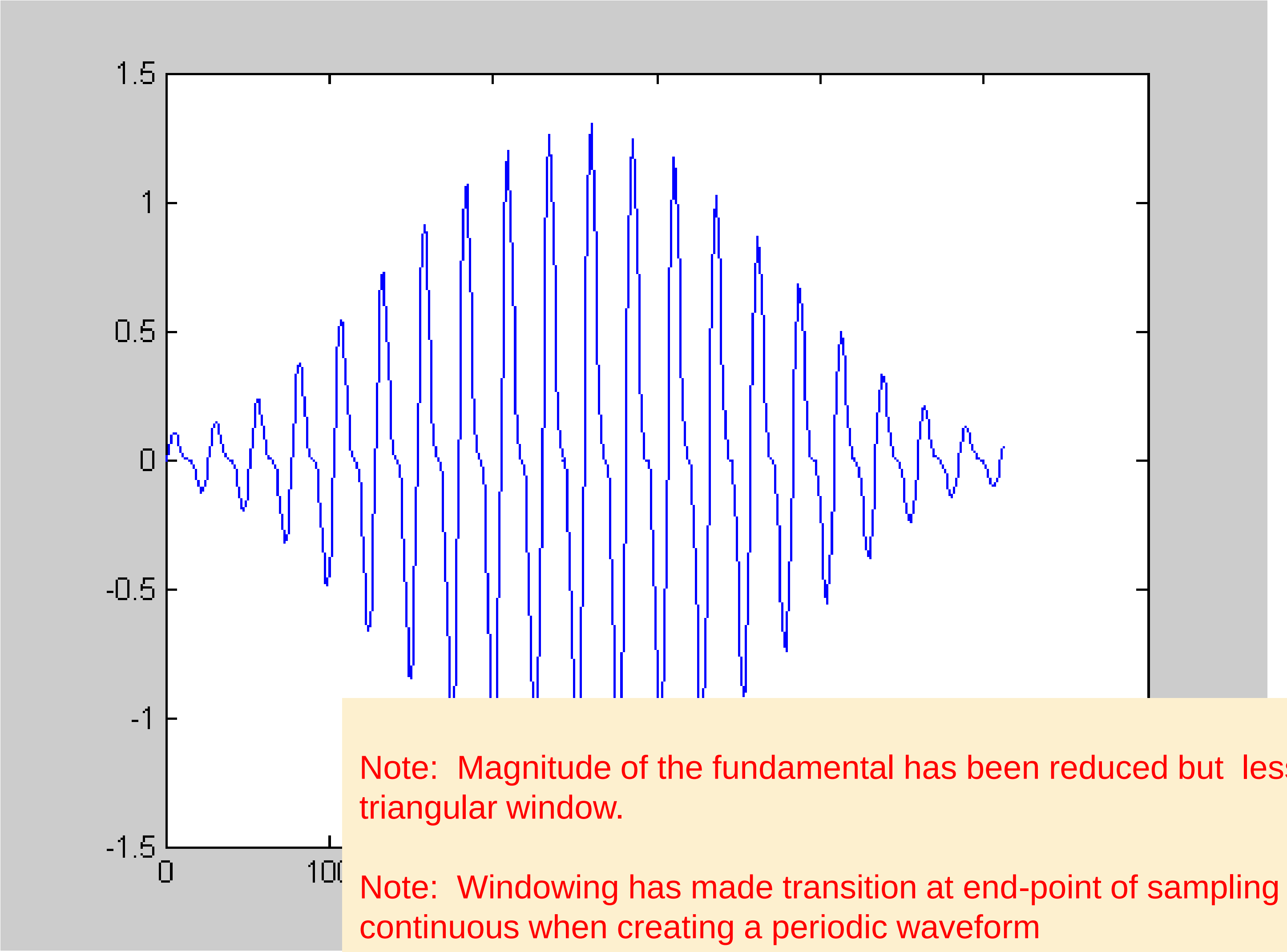
Colu

-77.

Note: Magnitude of the fundamental has been reduced but so have the skirting effects have also been reduced.

Note: Windowing has reduced energy in the signal but also made transition at end-point of sampling window continuous when creating a periodic waveform

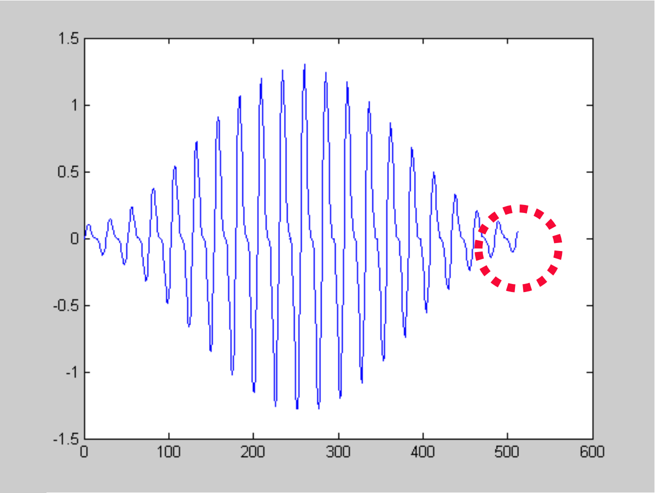
Hamming Window

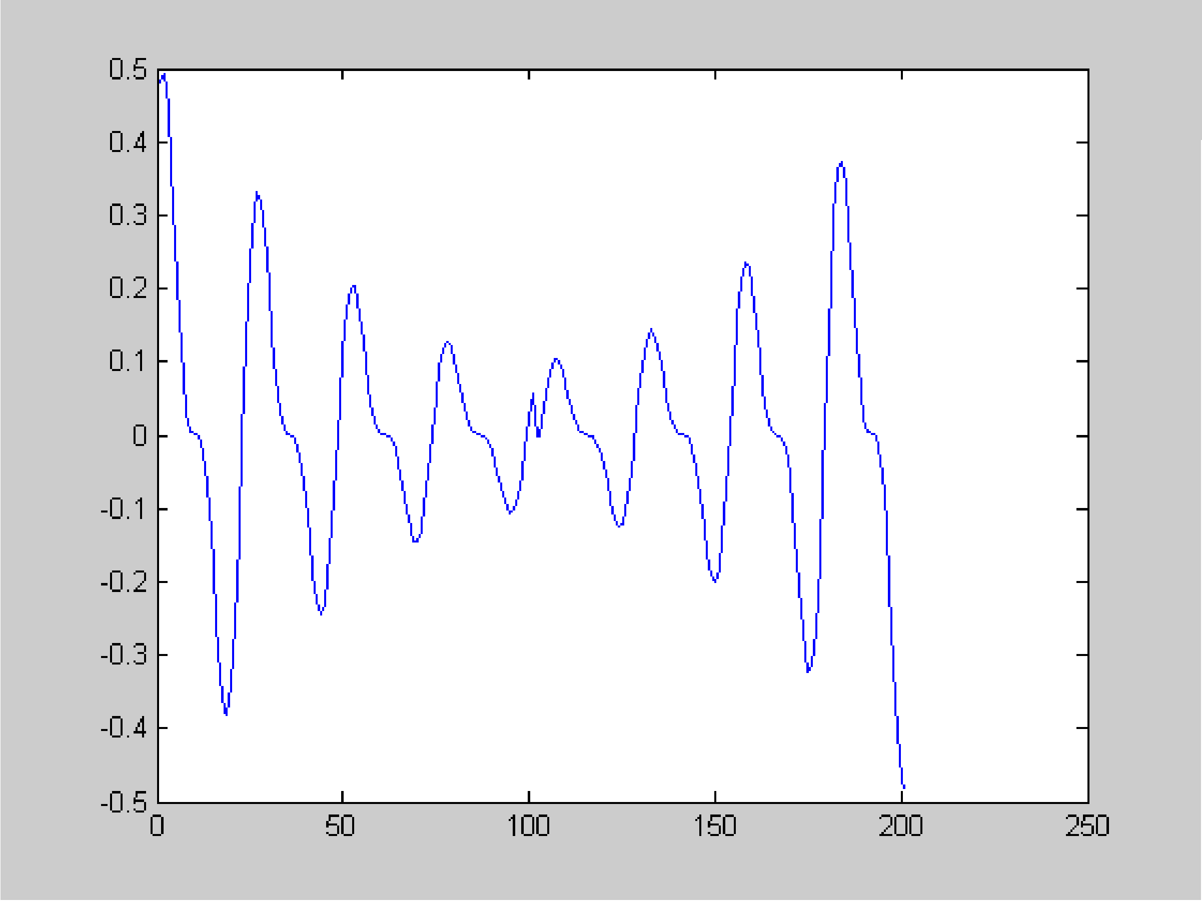


Note: Magnitude of the fundamental has been reduced but less than for triangular window.

Note: Windowing has made transition at end-point of sampling window continuous when creating a periodic waveform

Hamming Window





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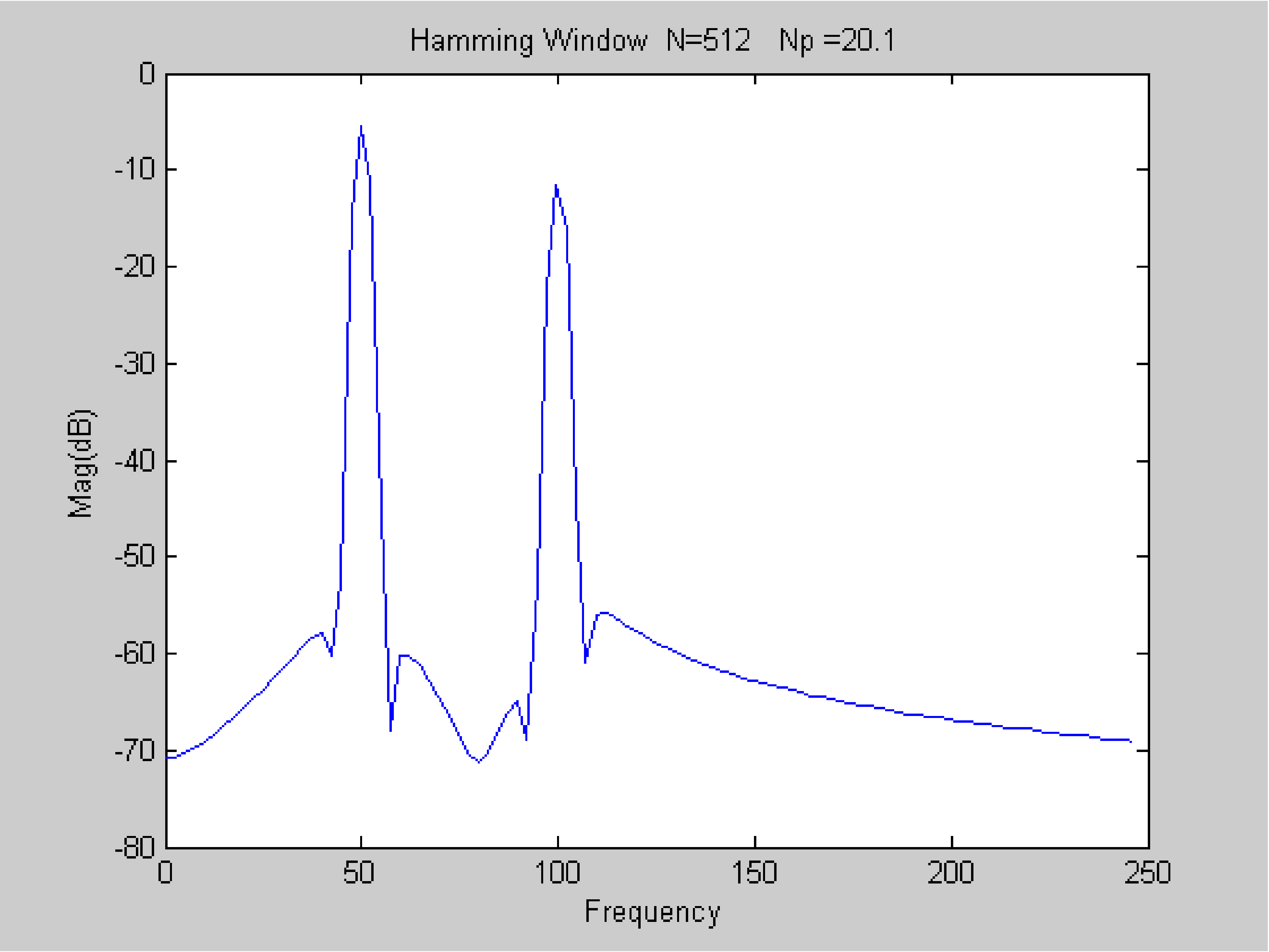
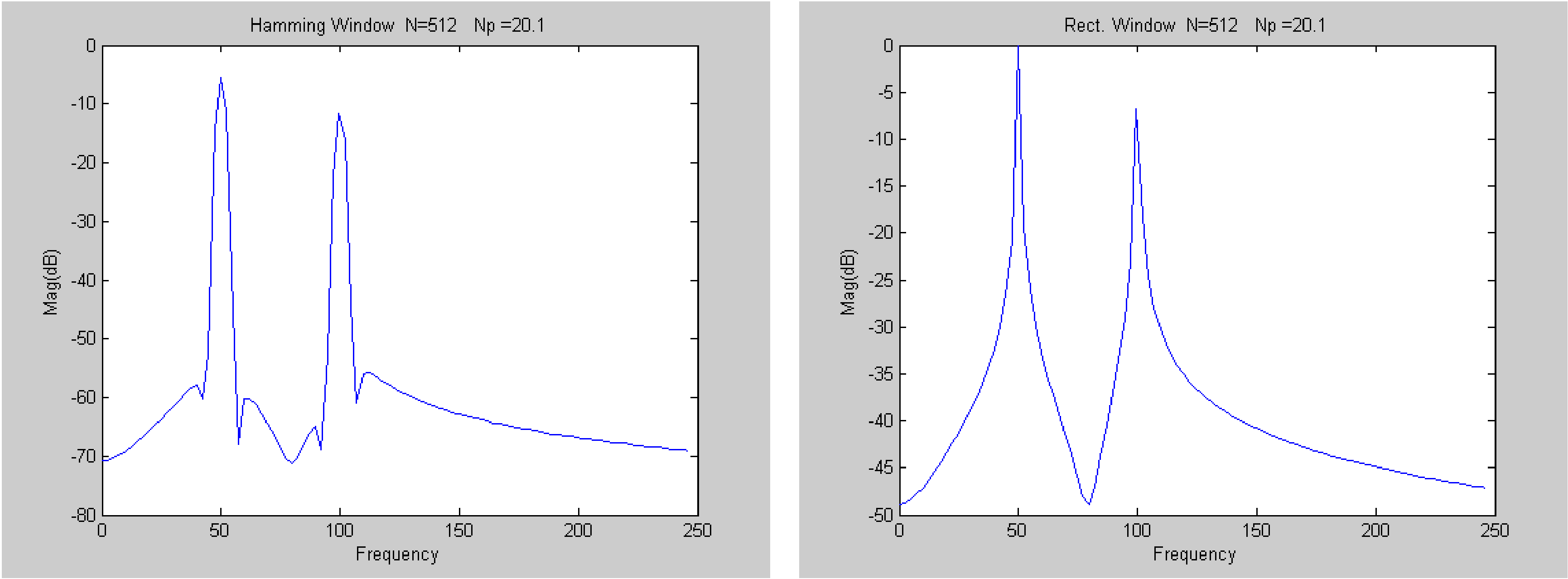


(zoomed in around fundamental)

Comparison with Rectangular Window



Note: Vertical axis are different

Hamming Window

Columns 1 through 7

-70.8278	-70.6955	-70.3703	-69.8555	-69.1502	-68.3632	-67.5133

Columns 8 through 14

-66.5945	-65.6321	-64.6276	-63.6635	-62.6204	-61.5590	-60.4199

Columns 15 through 21

-59.3204	-58.3582	-57.8735	-60.2994	-52.6273	-14.4702	-5.4343

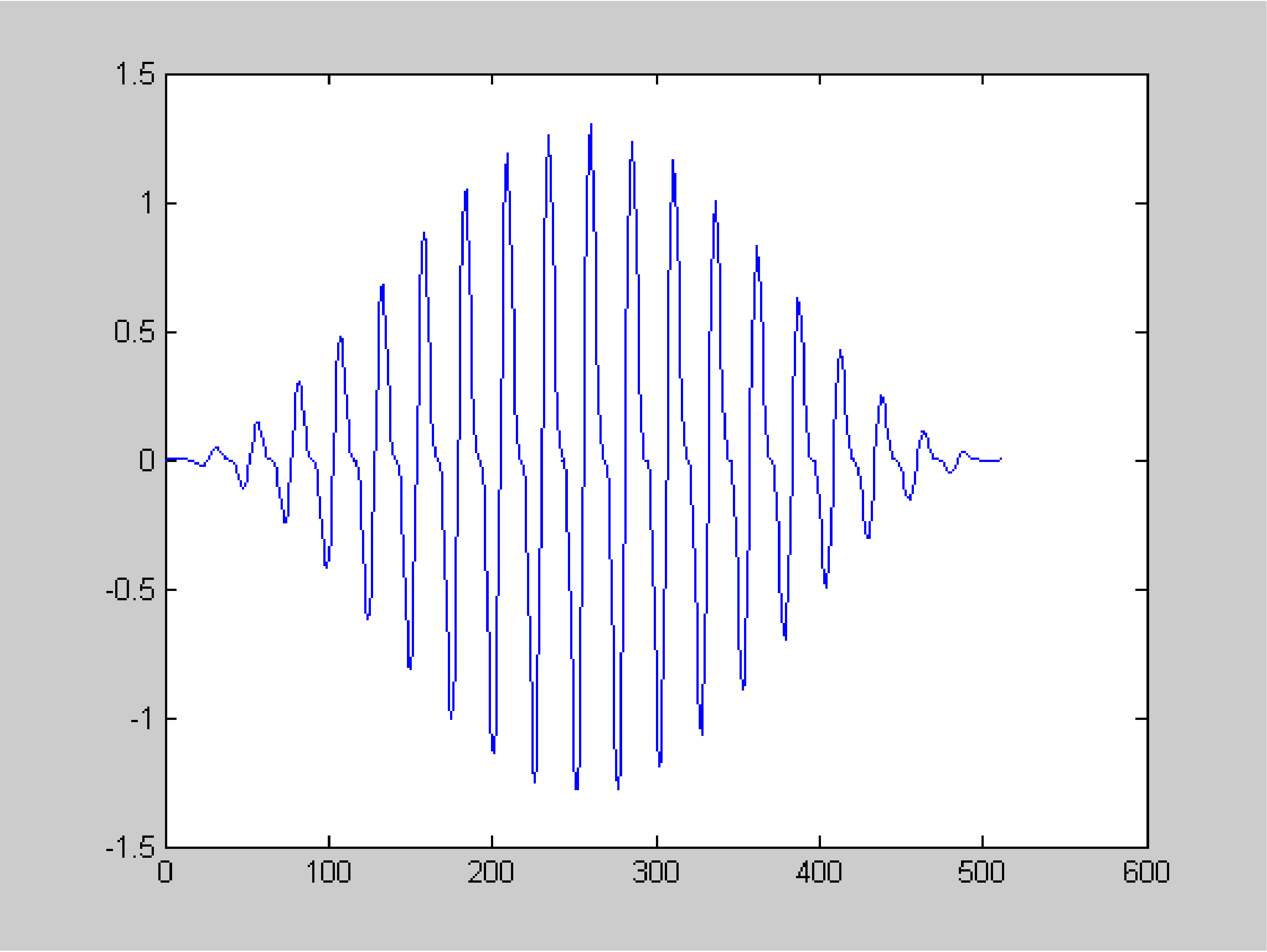
Columns 22 through 28

-11.2659	-45.2190	-67.9926	-60.1662	-60.1710	-61.2796	-62.7277

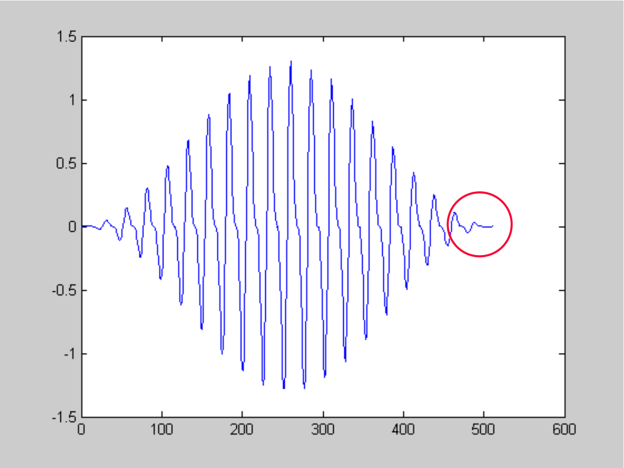
Columns 29 through 35

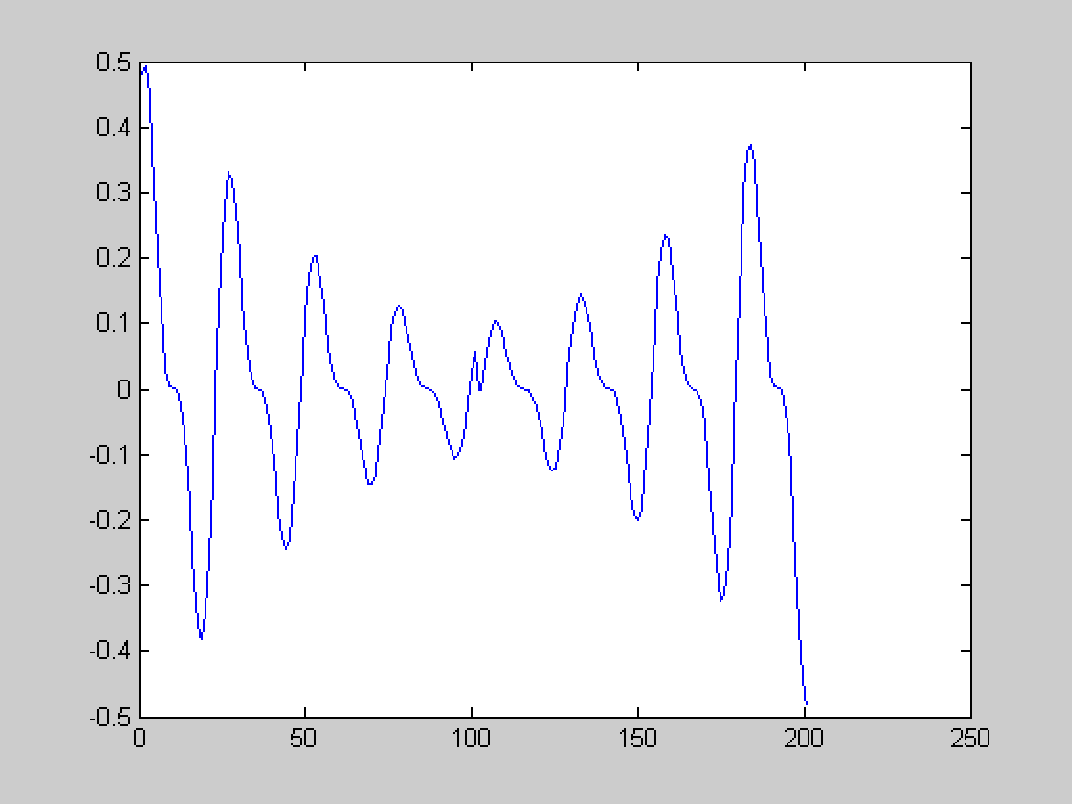
-64.3642	-66.2048	-68.2460	-70.1835	-71.1529	-70.2800	-68.1145

Hanning Window

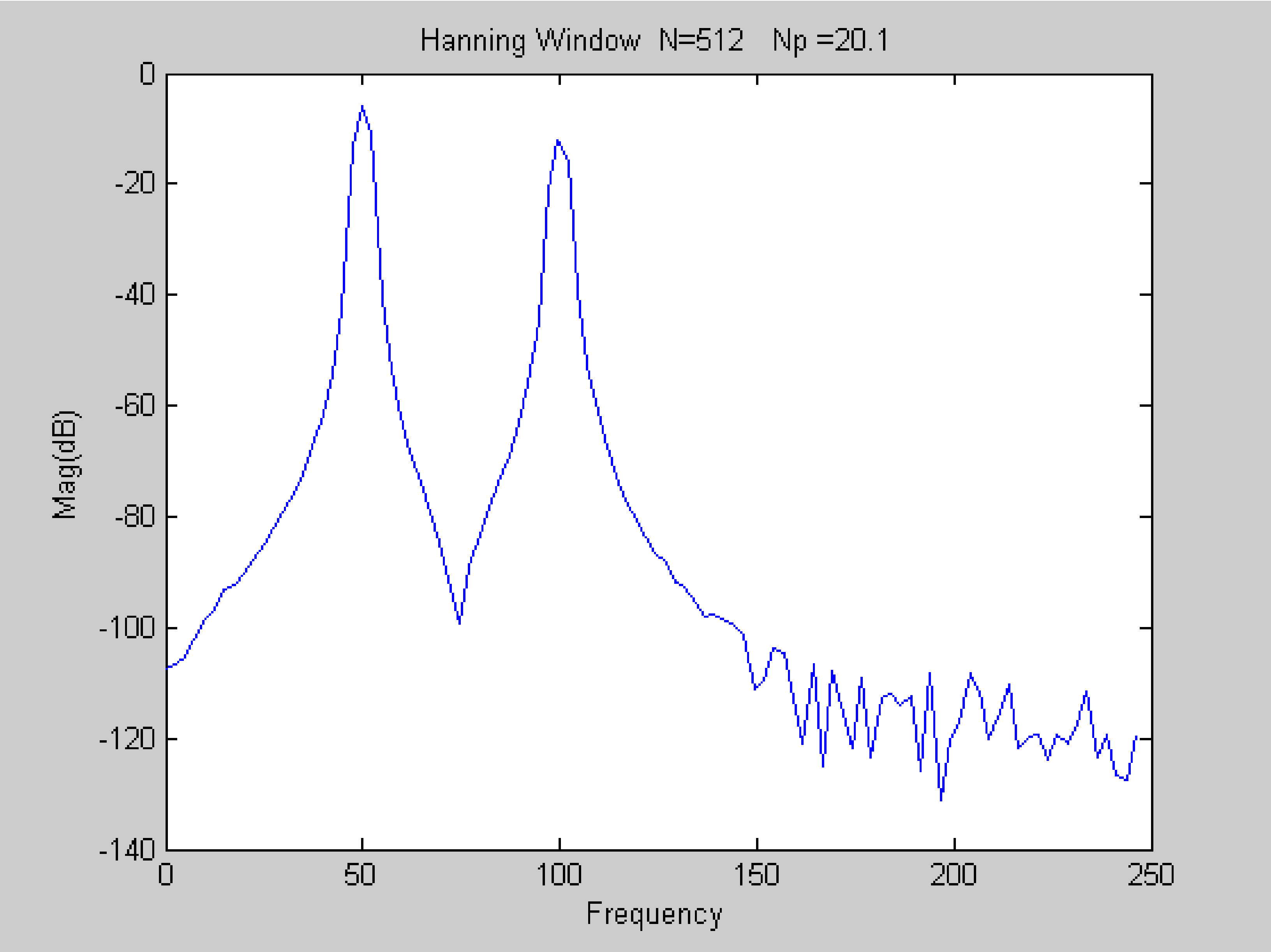


Hanning Window



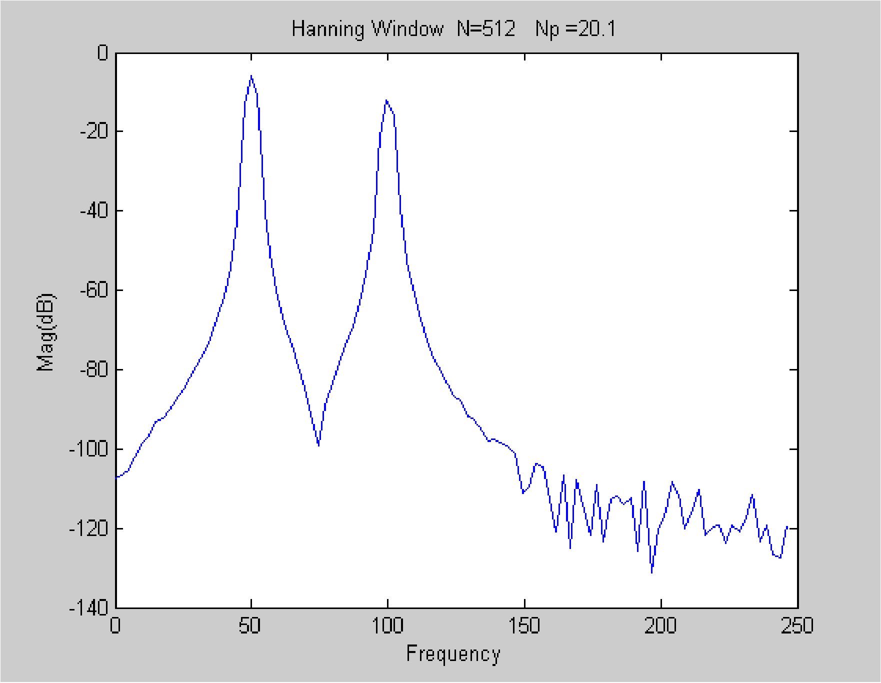


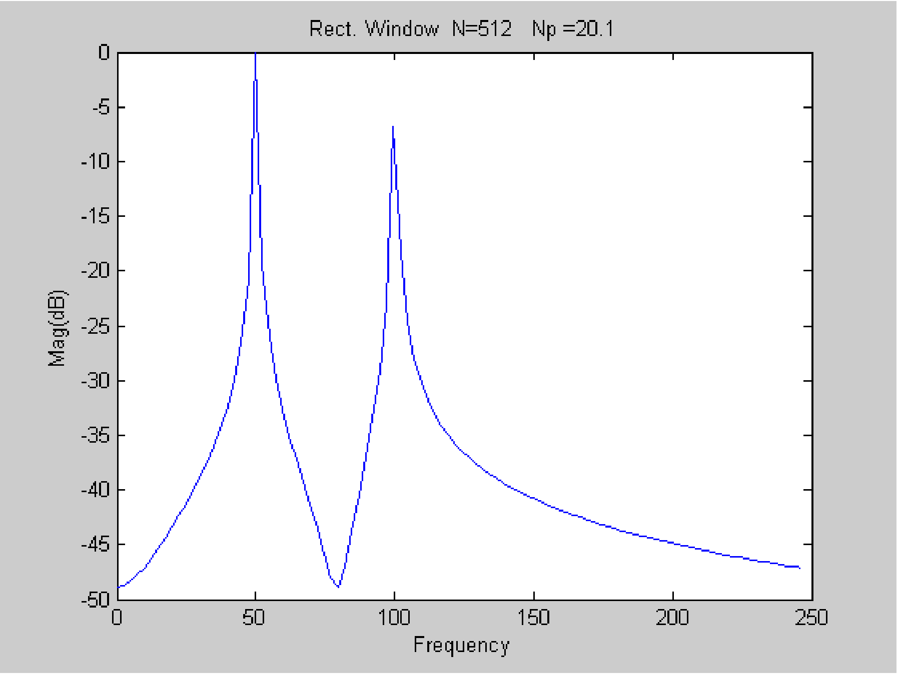
Spectral Response with Non-Coherent Sampling and Windowing



(zoomed in around fundamental)

Comparison with Rectangular Window





Note: Vertical axis are different

Hanning Window

Columns 1 through 7

-107.3123 -106.7939 -105.3421 -101.9488 -98.3043 -96.6522 -93.0343

Columns 8 through 14

-92.4519 -90.4372 -87.7977 -84.9554 -81.8956 -79.3520 -75.8944

Columns 15 through 21

-72.0479 -67.4602 -61.7543 -54.2042 -42.9597 -13.4511 -6.0601

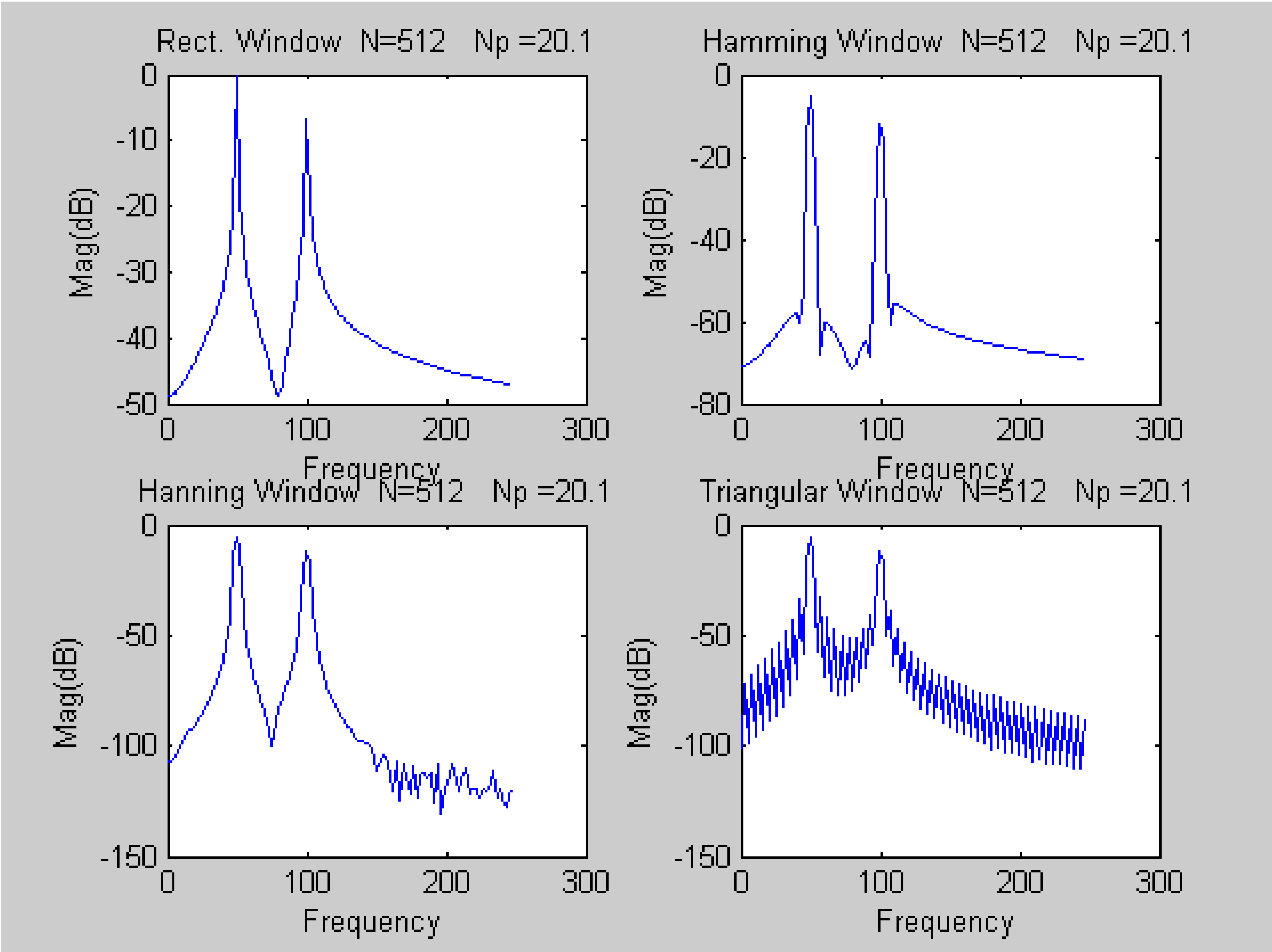
Columns 22 through 28

-10.8267 -40.4480 -53.3906 -61.8561 -68.3601 -73.9966 -79.0757

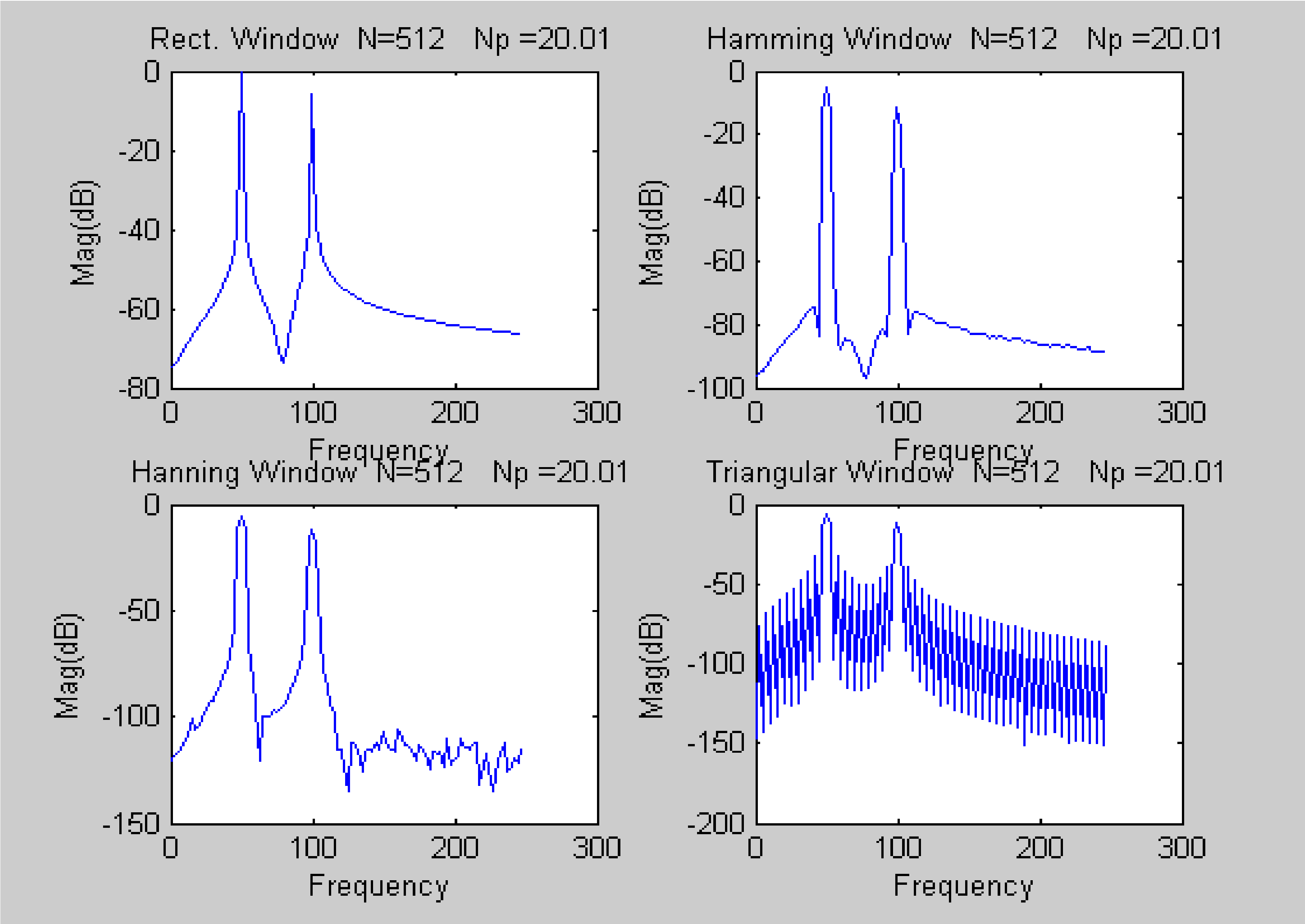
Columns 29 through 35

-84.4318 -92.7280 -99.4046 -89.0799 -83.4211 -78.5955 -73.9788

Comparison of 4 windows



Comparison of 4 windows

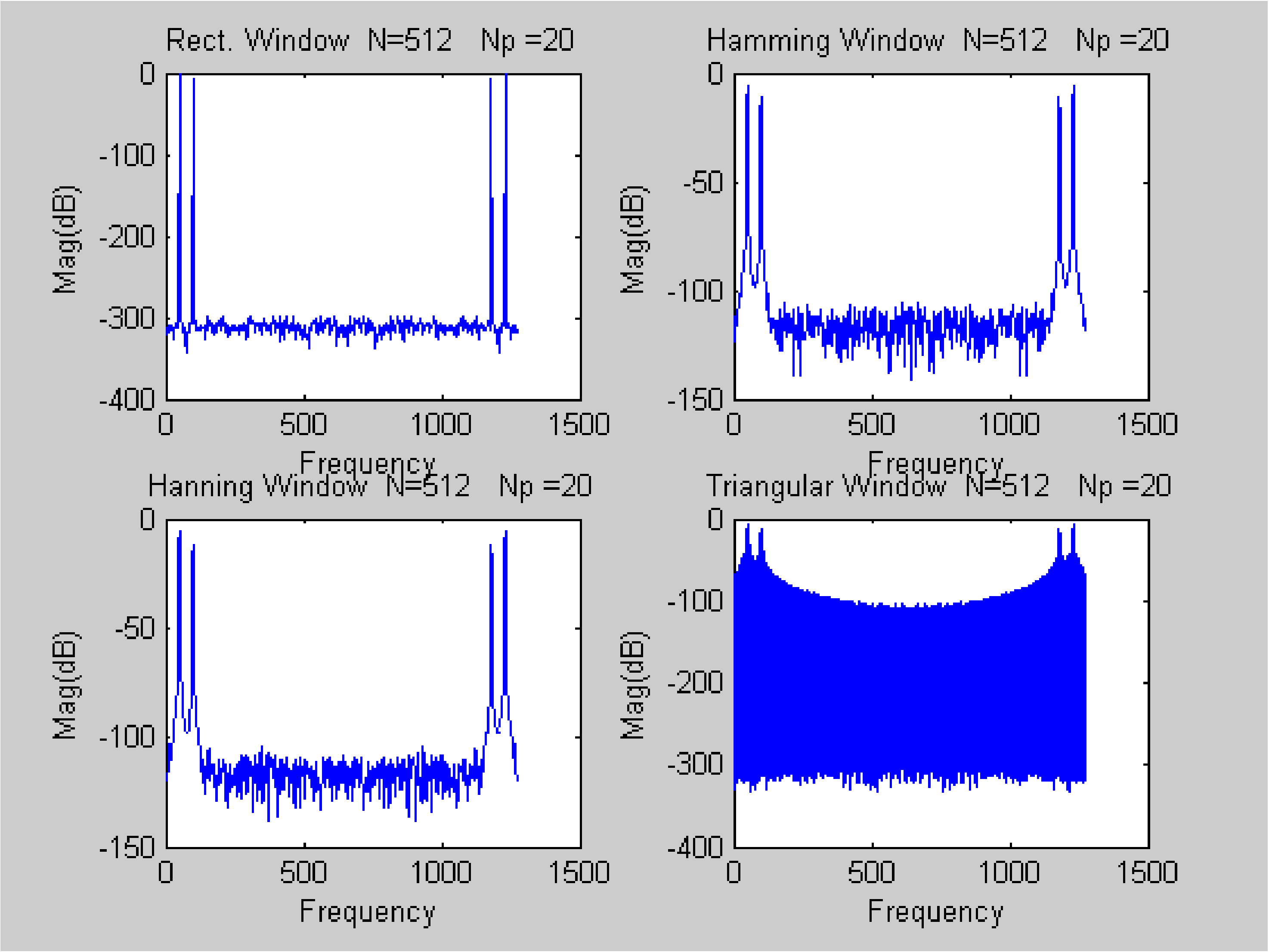


But windows can make things worse too!

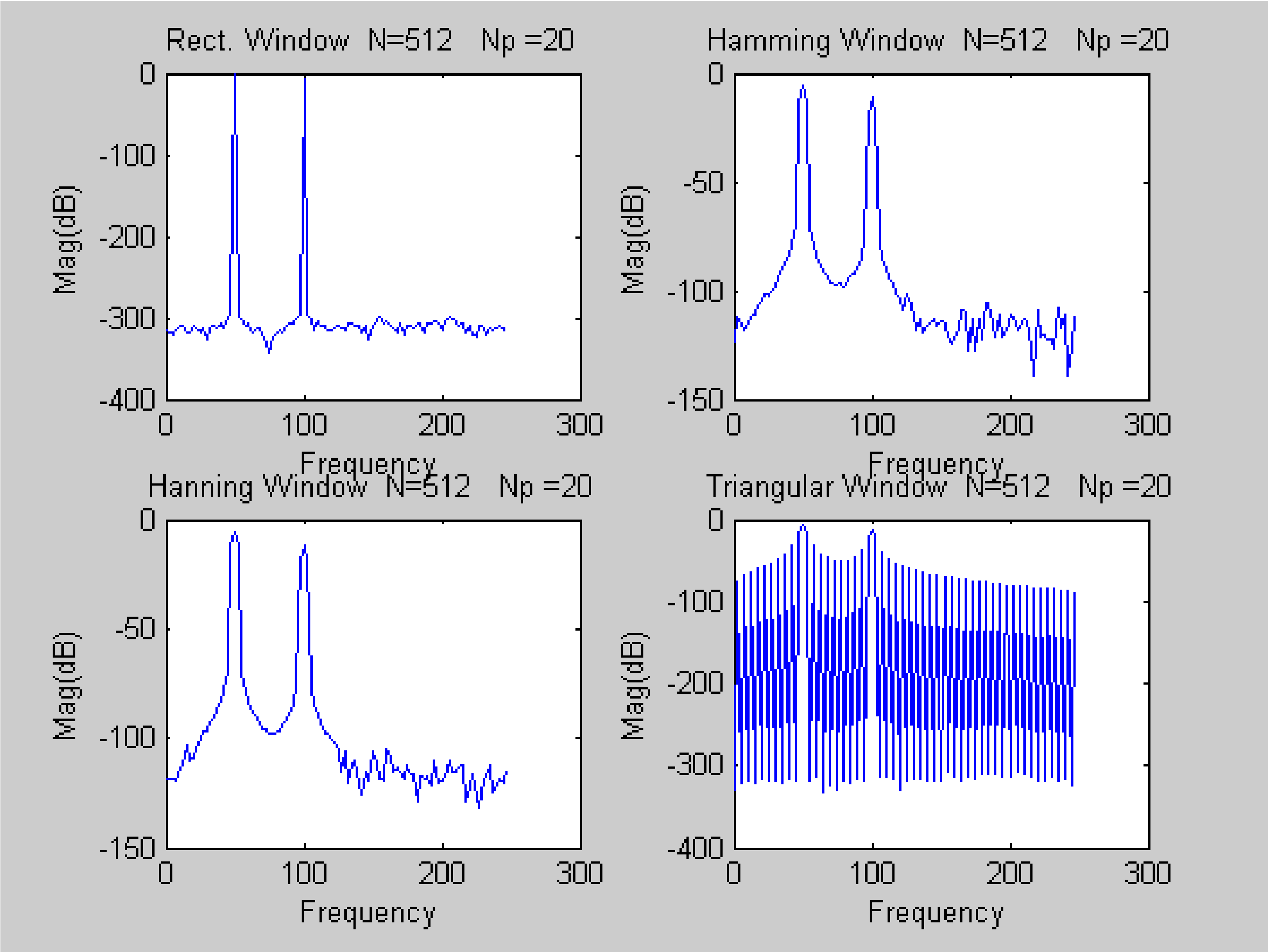
Consider situation where we really do have coherent sampling and a window is applied

fsig1=50Hz
fsig2=100Hz
N=512
Np=20

Comparison of 4 windows when sampling hypothesis are satisfied



Comparison of 4 windows



But windows can make things worse too!

Consider situation where we really do have coherent sampling and a window is applied

fsig1=50Hz
fsig2=100Hz
N=512
Np=20

And we do not really know how much worse thing can be!

Be careful about interpreting results obtained by using windowing to mitigate the non-coherent sampling problem !

Remember the hypothesis of the theorem relating the DFT, which is easy to calculate, to the Fourier Series coefficients!

Preliminary Observations about Windows

- Provide separation of spectral components
- Energy can be accumulated around spectral components
- Simple to apply
- Some windows work much better than others

But – windows do not provide dramatic improvement and can significantly degrade performance if sampling hypothesis are met

Issues of Concern for Spectral Analysis

An integral number of periods is critical for spectral analysis

Not easy to satisfy this requirement in the laboratory

Windowing can help but can hurt as well

Out of band energy can be reflected back into bands of interest

Characterization of CAD tool environment is essential

Spectral Characterization of high-resolution data converters requires particularly critical consideration to avoid simulations or measurements from masking real performance

Summary of time and amplitude quantization assessment

Time and amplitude quantization do not introduce harmonic distortion

Time and amplitude quantization do increase the noise floor

Performance Characterization of Data Converters

- Static characteristics
  - Resolution
  - Least Significant Bit (LSB)
  - Offset and Gain Errors
  - Absolute Accuracy
  - Relative Accuracy
  - Integral Nonlinearity (INL)
  - Differential Nonlinearity (DNL)
  - Monotonicity (DAC)
  - Missing Codes (ADC)
  - Quantization Noise
  - Low-f Spurious Free Dynamic Range (SFDR)
  - Low-f Total Harmonic Distortion (THD)
  - Effective Number of Bits (ENOB)
  - Power Dissipation

Quantization Noise

- DACs and ADCs generally quantize both amplitude and time
- If converting a continuous-time signal (ADC) or generating a desired continuous-time signal (DAC) these quantizations cause a difference in time and amplitude from the desired signal – this difference is termed “noise”.
- First a few comments about Noise

What is Noise in a data converter?

Noise is a term applied to some nonideal effects of a data converter

Precise definition of noise is probably not useful

Some differences in views about what nonideal characteristics of a data converter should be referred to as noise

Types of noise:

- Random noise due to movement of electrons in electronic circuits (resistors and active devices)
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits
  - Quantization noise
  - Sample Jitter
  - Harmonic Distortion

Noise

Some major types of noise:

- Random noise due to movement of electrons in electronic circuits
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits

All of these types of noise are present in data converters and are of concern when designing most data converters

Can not eliminate any of these noise types but with careful design can manage their effects to certain levels

Noise (in particular the random noise) is often the major factor limiting the ultimate performance potential of many if not most data converters

Noise

Some major types of noise:

- Random noise due to movement of electrons in electronic circuits
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits

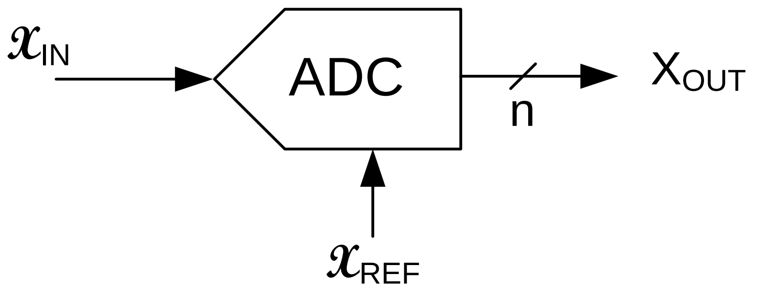
Quantization noise is a significant component of this noise in ADCs and DACs and is present even if the ADC or DAC is ideal

Will now investigate quantization noise

Quantization Noise in ADC

(same concepts apply to DACs)

Consider an Ideal ADC with first transition point at $0.5X_{LSB}$ where XLSB is determined by the bits of resolution of the ADC



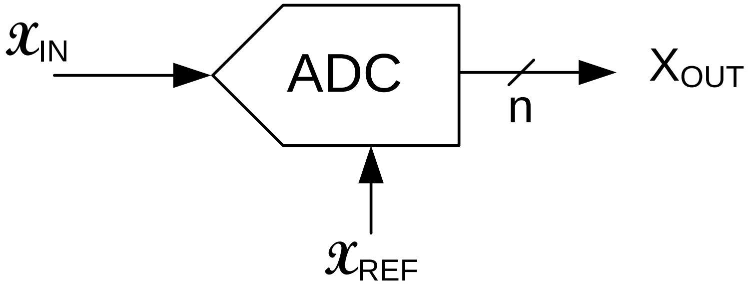
Consider an input that is a low frequency sawtooth waveform of period T that goes from 0 to X_{REF} that is sampled very fast so that the digital output always represents a quantized version of the input.

Draw here:

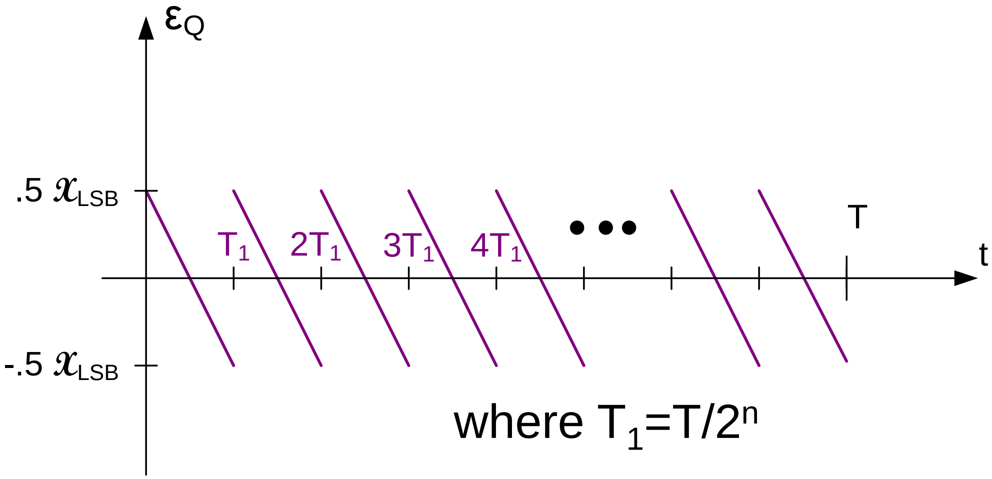
Quantization Noise in ADC

(same concepts apply to DACs)

Consider an Ideal ADC with first transition point at $0.5X_{LSB}$



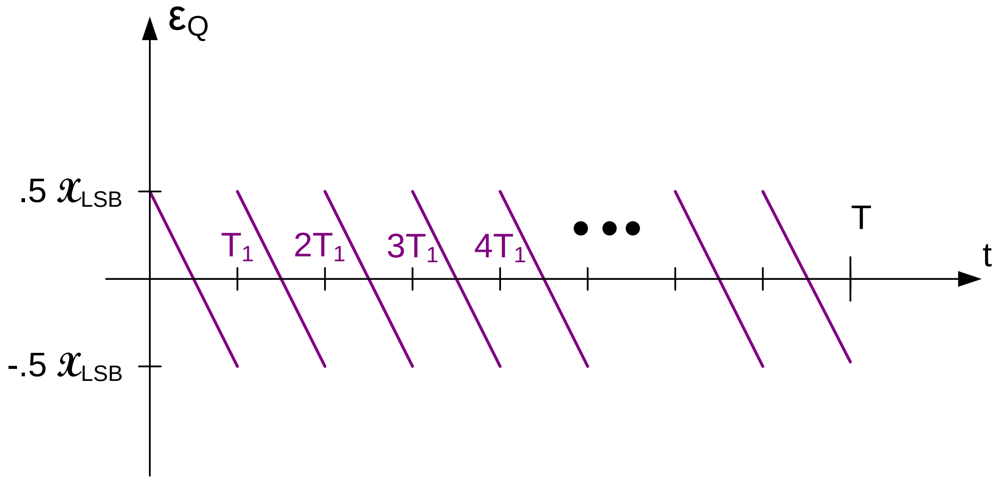
If the input is a low frequenocy sawtooth waveform of period T that goes from 0 to X_{REF} and if sampling is very fast so that the digital output always represents a quantized version of the input, the error signal in the time domain will be:



where $T_1 = T/2^n$

This time-domain waveform is termed the Quantization Noise for the ADC with a sawtooth (or triangular) input

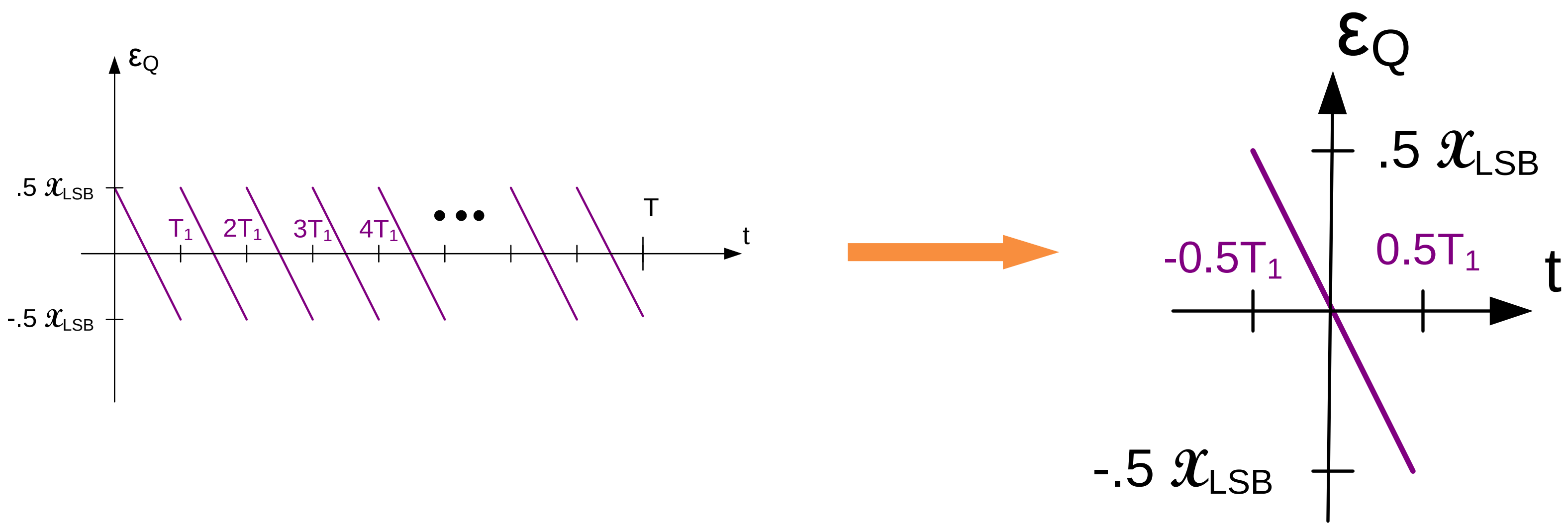
Quantization Noise in ADC



For large n, this periodic waveform behaves much like a random noise source that is uncorrelated with the input and can be characterized by its RMS value which can be obtained by integrating over any interval of length T_1. For notational convenience, shift the waveform by $T_1/2$ units

$$E_{RMS} = \sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} \varepsilon_Q^{2}(t)\,dt}$$

Quantization Noise in ADC



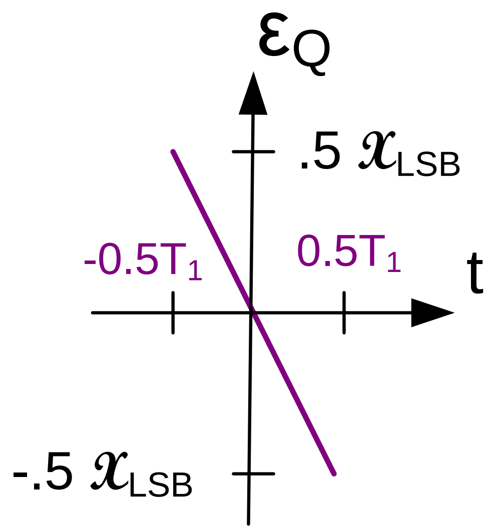
$$E_{RMS} = \sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} \varepsilon_Q{}^2(t)\,dt}$$

In this interval, ε_Q can be expressed as

$$\varepsilon_Q(t) = -\left(\frac{X_{LSB}}{T_1}\right)t$$

Quantization Noise in ADC

$$E_{RMS} = \sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} \varepsilon_Q{}^2(t)\,dt}$$



$$\varepsilon_Q(t) = -\left(\frac{X_{LSB}}{T_1}\right)t$$

$$E_{RMS} = \sqrt{\frac{1}{T_1}\int_{-T_1/2}^{T_1/2} \left(-\frac{x_{LSB}}{T_1}\right)^2 t^2\,dt}$$

$$E_{RMS} = x_{LSB}\sqrt{\frac{1}{T_1^3}\left.\frac{t^3}{3}\right|_{-T_1/2}^{T_1/2}}$$

$$E_{RMS} = \frac{x_{LSB}}{\sqrt{12}}$$

Quantization Noise in ADC

$$E_{RMS} = \frac{x_{LSB}}{\sqrt{12}}$$

The signal to quantization noise ratio (SNR) can now be determined. Since the input signal is a sawtooth waveform of period T and amplitude X_{REF}, it follows by the same analysis that it has an RMS value of

$$x_{RMS} = \frac{x_{REF}}{\sqrt{12}} = \frac{2^n X_{LSB}}{\sqrt{12}}$$

Thus the SNR is given by

$$SNR = \frac{x_{RMS}}{E_{RMS}} = \frac{2^n x_{LSB}}{x_{LSB}} = 2^n$$

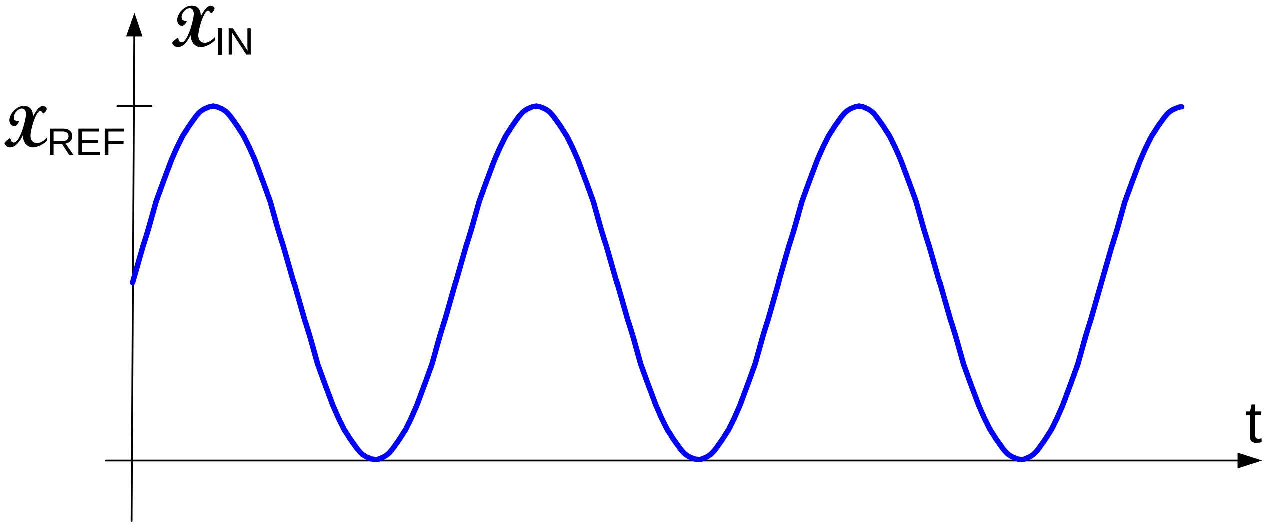
or, in dB,

$$SNR_{dB} = 20(n \bullet \log 2) = 6.02n$$

Note: dB subscript often neglected when not concerned about confusion

Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?

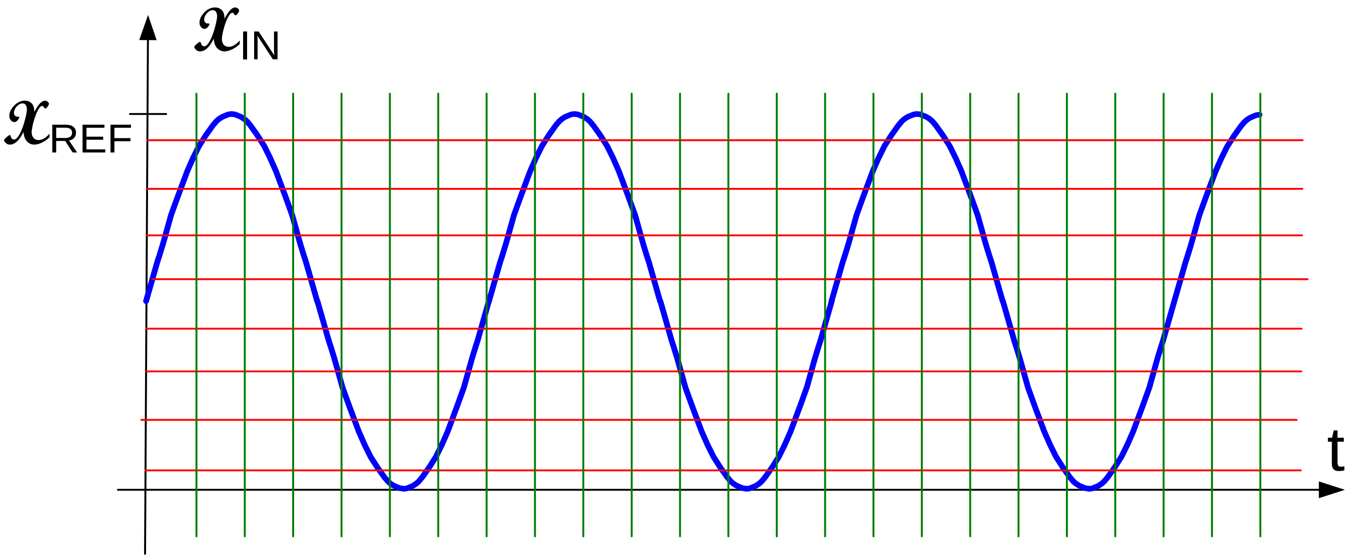


For full-scale sawtooth (or triangular input) $SNR = 20(n \bullet \log 2) = 6.02n$

For full-scale sinusoidal input SNR = ? ? ?

Quantization Noise in ADC

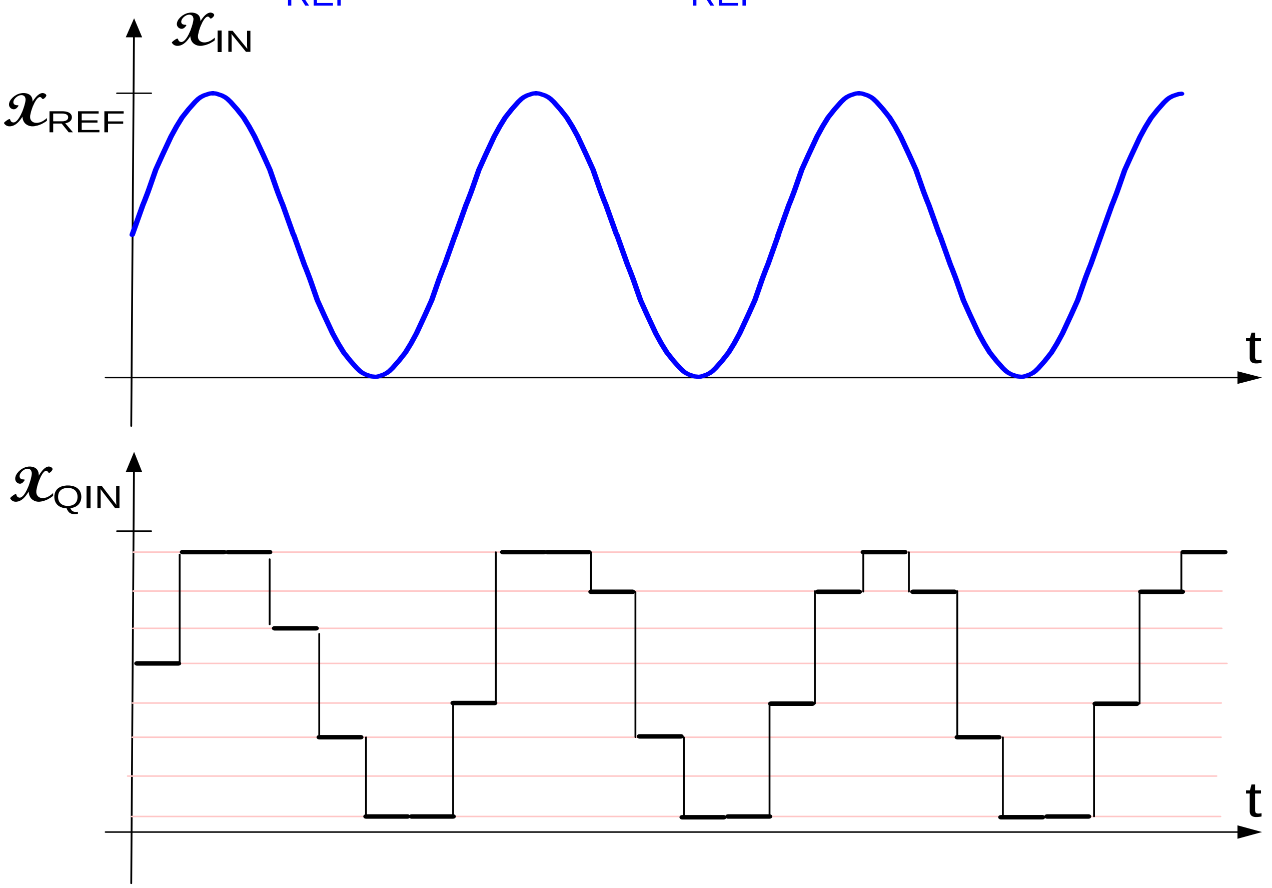
How does the SNR change if the input is a sinusoid that goes from 0 to x_{REF} centered at $x_{REF}/2$?



Time and amplitude quantization points

Quantization Noise in ADC

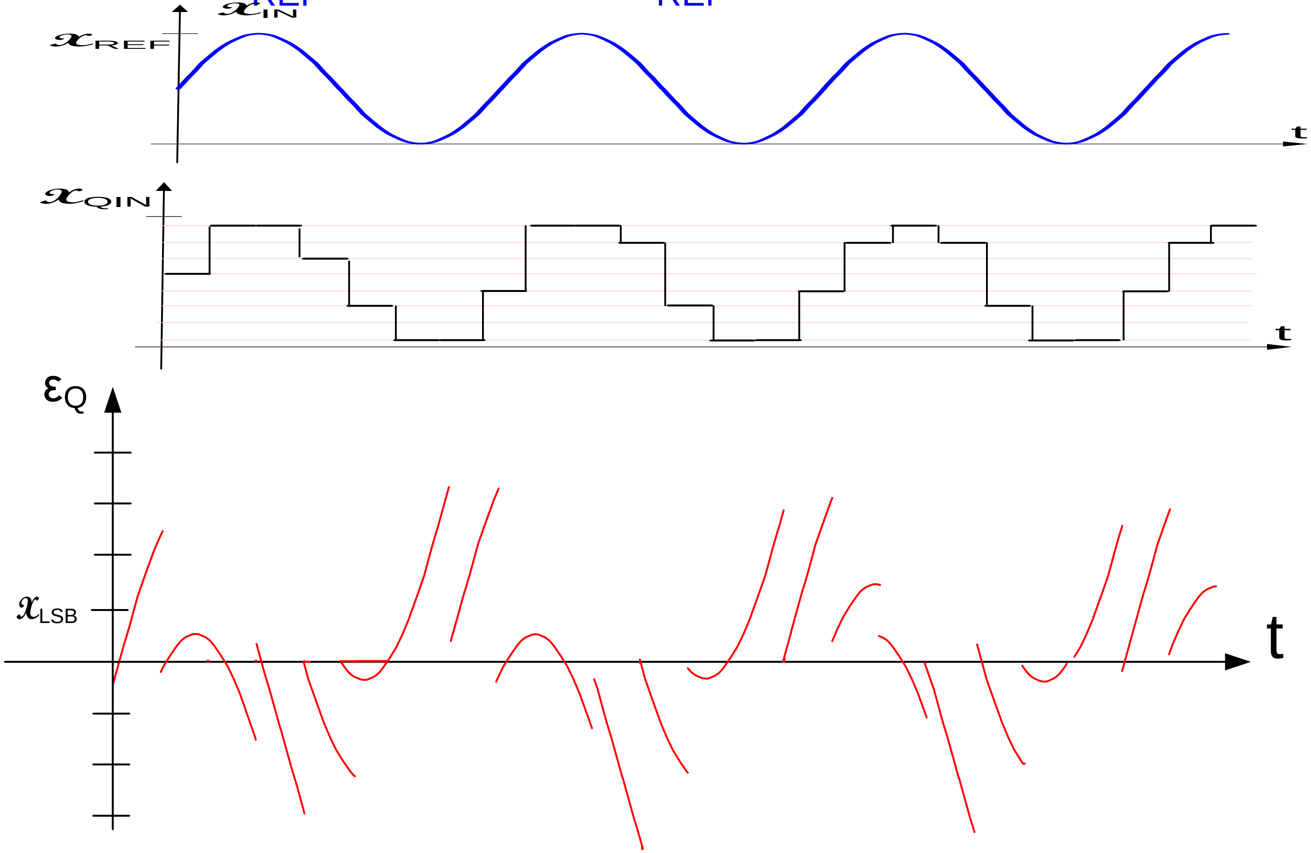
How does the SNR change if the input is a sinusoid that goes from 0 to x_{REF} centered at $x_{REF}/2$?



Time and Amplitude Quantized Waveform

Quantization Noise in ADC

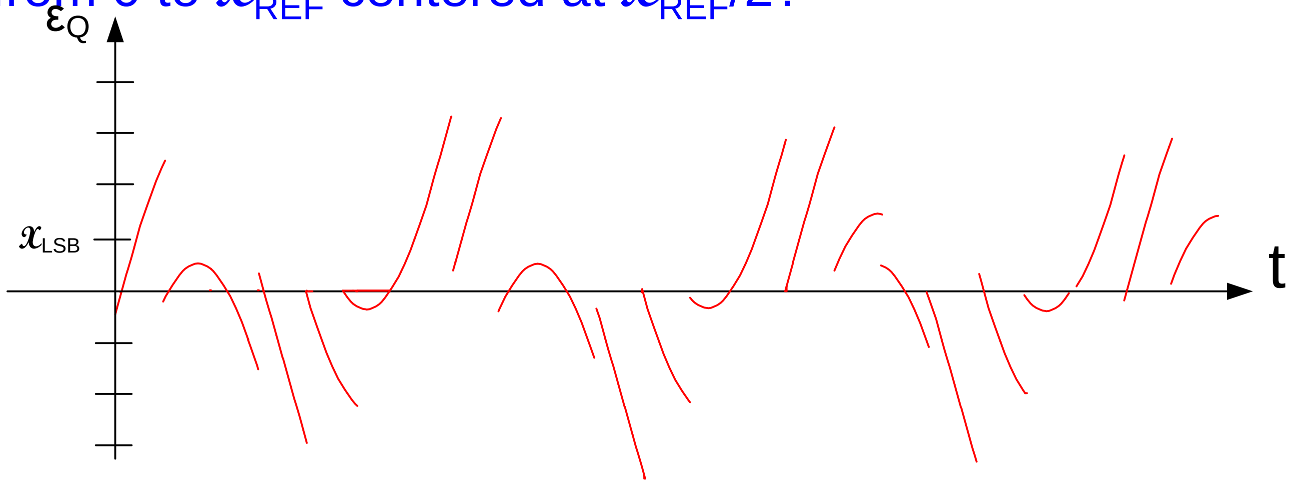
How does the SNR change if the input is a sinusoid that goes from 0 to x_{REF} centered at $x_{REF}/2$?



Error waveform

Quantization Noise in ADC

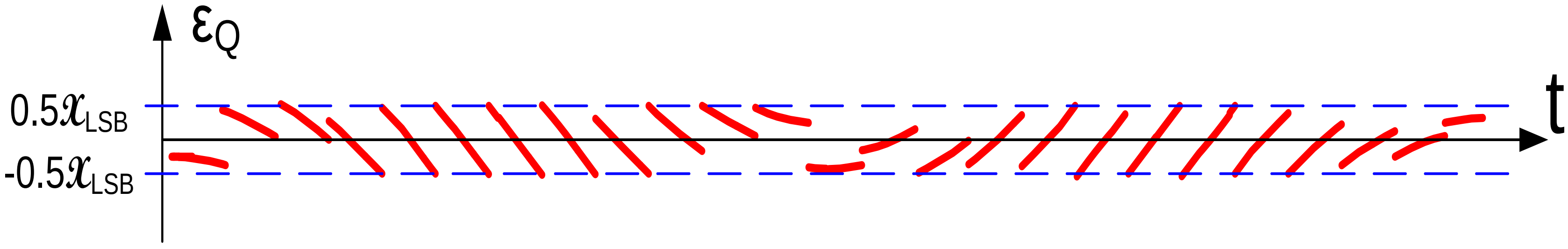
How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?



- Appears to be highly uncorrelated with input even though deterministic
- Mathematical expression for ε_Q very messy
- Excursions exceed X_{LSB}
- For low frequency inputs and higher resolution, at any time, errors are approximately uniformly distributed between $-X_{LSB}/2$ and $X_{LSB}/2$
- Analytical form for ε_{QRMS} essentially impossible to obtain from $\varepsilon_Q(t)$

Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?



For low f_{SIG}/f_{CL} ratios, bounded by $\pm 0.5\ X_{LSB}$ and at any point in time, behaves almost as if a uniformly distributed random variable

$$\varepsilon_Q \sim U[-0.5X_{LSB},\ 0.5X_{LSB}]$$

Quantization Noise in ADC

Recall:

If the random variable f is uniformly distributed in the interval [A,B]
f : U[A,B]   then the mean and standard deviation of f are given by

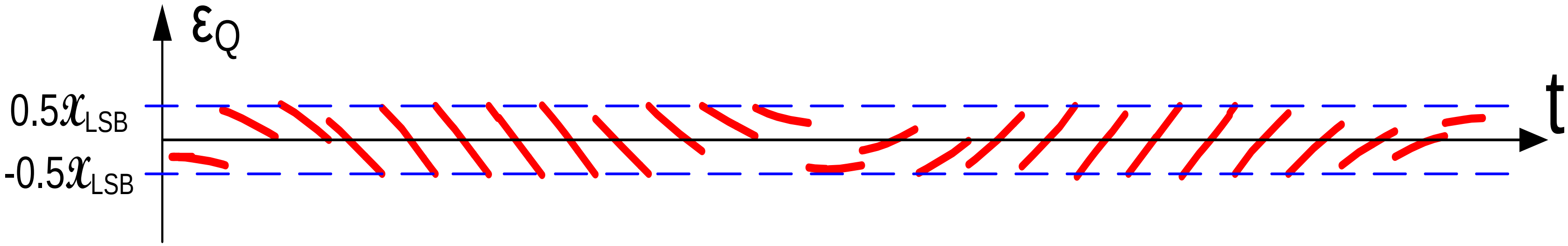
$$\mu_f = \frac{A+B}{2} \qquad \sigma_f = \frac{B-A}{\sqrt{12}}$$

Theorem: If n(t) is a random process and <n(kT_S)> is a sequence of samples of n(t) then for large T/T_S,

$$V_{RMS} = \sqrt{\frac{1}{T}\int_{t_1}^{t_1+T} n^2(t)\,dt} = \sqrt{\sigma^2_{n(kT_S)} + \mu^2_{n(kT_S)}}$$

Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?



$$\varepsilon_Q \sim U[-0.5X_{LSB}, 0.5X_{LSB}]$$

$$\mu_{\varepsilon_Q} = \frac{A+B}{2} = 0 \qquad \sigma_f = \frac{B-A}{\sqrt{12}} = \frac{X_{LSB}}{\sqrt{12}}$$

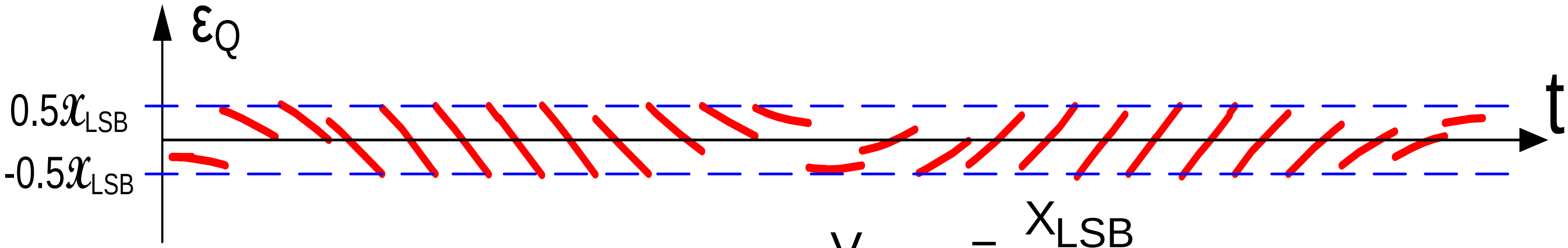
$$V_{RMS} = \sqrt{\frac{1}{T}\int_{t_1}^{t_1+T} n^2(t)\,dt} = \sqrt{\sigma_n^2 + \mu_n^2}$$

$$V_{RMS} = \sigma_{\varepsilon_Q} = \frac{X_{LSB}}{\sqrt{12}}$$

Note this is the same RMS noise that was present with a triangular input

Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $\mathcal{X}_{REF}$ centered at $\mathcal{X}_{REF}/2$?



$$V_{RMS} = \frac{X_{LSB}}{\sqrt{12}}$$

But $$V_{INRMS} = \left(\frac{X_{REF}}{2}\right)\frac{1}{\sqrt{2}}$$

Thus obtain

$$SNR = \frac{\dfrac{X_{REF}}{2\sqrt{2}}}{\dfrac{X_{LSB}}{\sqrt{12}}} = 2^n\sqrt{\frac{3}{2}}$$

Finally, in db,

$$SNR_{dB} = 20\log\left(2^n\sqrt{\frac{3}{2}}\right) = 6.02\,n + 1.76$$

ENOB based upon Quantization Noise Reference

Different factors can cause the SNR or SNDR of an ADC to not be ∞

- Quantization effects
- Device noise
- Interference Noise
- Nonlinear distortion
- Signal amplitude
- Jitter
- Computation errors
- …….

It is often useful to consider how an ADC performs from a SNR or SNDR viewpoint relative to how it would perform if only quantization effects (which are unavoidable) for an otherwise ideal ADC are present

An ENOB relative to an otherwise ideal ADC is often used as a metric for assessing SNR or SNDR performance

For example, consider a 14-bit ADC with a full-signal sinusoidal input that has quantization noise of $\frac{X_{LSB}}{\sqrt{12}} = 0.29X_{LSB}$, device noise with an RMS value of $2X_{LSB}$ and interference noise of 5 X_{LSB}. The total noise is then $5.4X_{LSB}$. Thus its SNR is equivalent to that of a much lower resolution ADC that has only quantization noise present. The resolution of that lower resolution ADC would be termed the ENOB relative to a Quantization Noise Only data converter

ENOB based upon Quantization Noise Reference

$$SNR_{dB} = 6.02\,n + 1.76$$

Solving for n, obtain

$$ENOB = \frac{SNR_{dB} - 1.76}{6.02}$$

Note: could have used the SNR_{dB} for a triangle input and would have obtained the expression

$$ENOB = \frac{SNR_{dB}}{6.02}$$

But the earlier expression is more widely used when specifying the ENOB based upon the noise level present in a data converter

What is ENOB (using standard sinusoidal reference definition) if only quantization noise present with a full-scale sinusoidal input?

ENOB based upon Quantization Noise Reference

$$SNR_{dB} = 6.02\,n + 1.76$$

Solving for n, obtain

$$ENOB = \frac{SNR_{dB} - 1.76}{6.02}$$

Note: could have used the SNR_{dB} for a triangle input and would have obtained the expression

$$ENOB = \frac{SNR_{dB}}{6.02}$$

But the earlier expression is more widely used when specifying the ENOB based upon the noise level present in a data converter

What is ENOB if only quantization noise present with a full-scale sinusoidal input?

$$ENOB = \frac{SNR_{dB} - 1.76}{6.02} = \frac{6.02n + 1.76 - 1.76}{6.02} = n$$

ENOB based upon Quantization Noise

For very low resolution levels, the assumption that the quantization noise is uncorrelated with the signal is not valid and the ENOB expression will cause a modest error

from van de Plassche (p13)

$$SNR_{corr} \cong \left(2^n - 2 + \frac{4}{\pi}\right)\sqrt{\frac{3}{2}}$$

Res (n)	SNR_{corr}	SNR
1	3.86	7.78
2	12.06	13.8
3	19.0	19.82
4	25.44	25.84
5	31.66	31.86
6	37.79	37.88
8	49.90	49.92
10	61.95	61.96

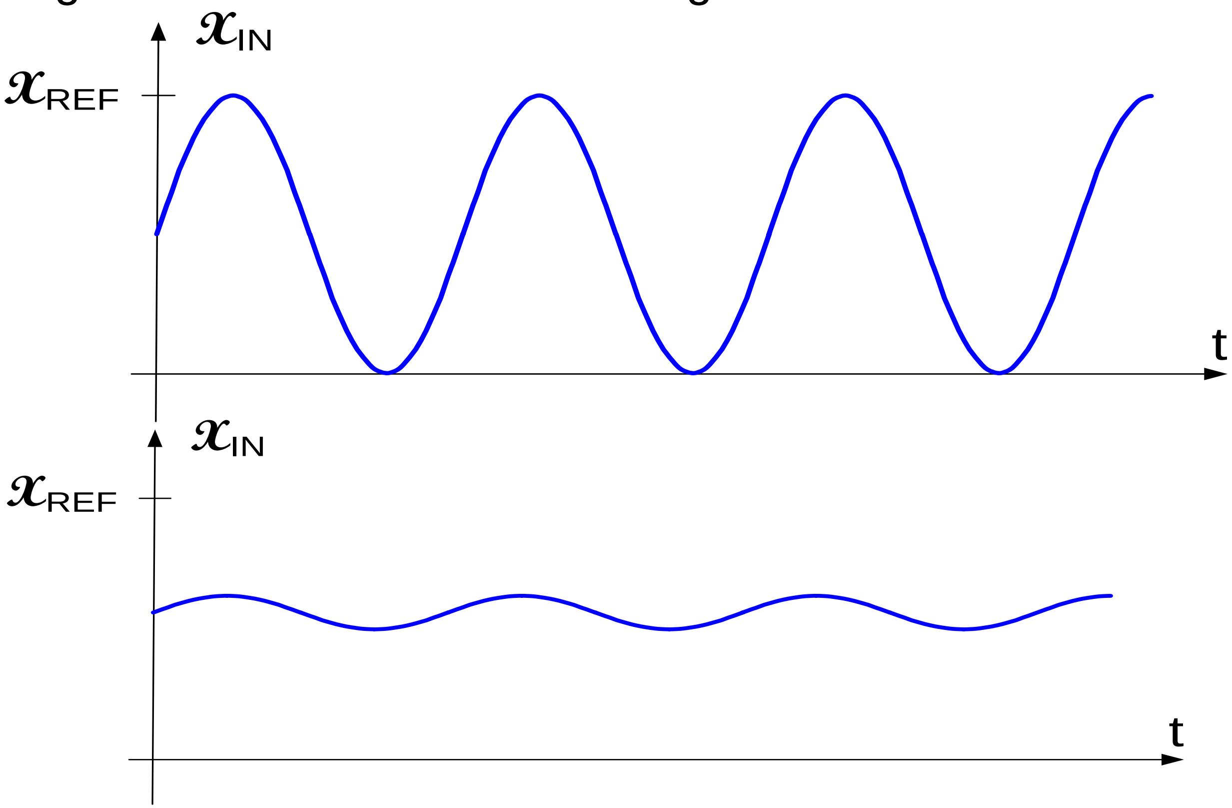
$SNR = 6.02\ n + 1.76$

Table values in dB

Almost no difference for $n \geq 3$

Quantization Noise

Effects of quantization noise can be very significant, even at high resolution, when signals are not of maximum magnitude

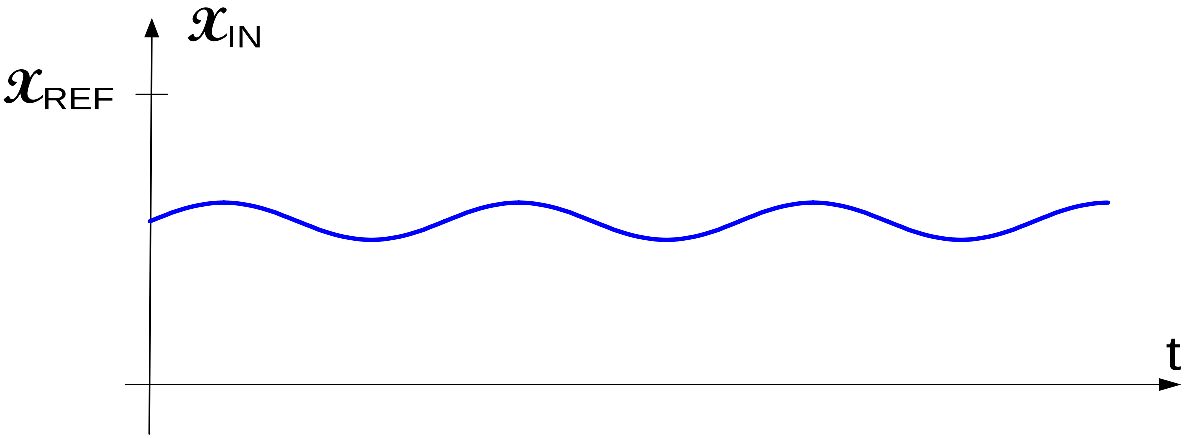


Quantization noise remains constant but signal level is reduced

The desire to use a data converter at a small fraction of full range is one of the major reasons high resolution is required

Quantization Noise

Effects of quantization noise can be very significant, even at high resolution, when signals are not of maximum magnitude



Quantization Noise

Example: If a 14-bit audio output is derived from a DAC designed for providing an output of 100W but the normal listening level is at 50mW, what is the SNR due to quantization noise at maximum output and at the normal listening level? What is the ENOB of the audio system when operating at 50mW?

At 100W output, SNR=6.02n+1.76 = 86.04dB

$$\frac{V^2}{R_L} = 100W \qquad \frac{V_1^2}{R_L} = 50mW \qquad V_1 = \frac{V}{44.7}$$

$$20\log_{10}V_1 = 20\log_{10}V - 20\log_{10}44.7 = -33dB$$

At 50mW output, SNR reduced by 33dB to 53.04dB

$$ENOB = \frac{SNR_{dB} - 1.76}{6.02} = \frac{53.04 - 1.76}{6.02} = 8.51$$

Note the dramatic reduction in the effective resolution of the DAC when operated at only a small fraction of full-scale.

ENOB Summary

Resolution:

$$\text{ENOB} = \frac{\log_{10} N_{ACT}}{\log_{10} 2} = \log_2 N_{ACT}$$

INL:

$$\text{ENOB} = n_R - \log_2(\upsilon) - 1$$

n_R specified res, v INL in LSB

DNL:

$$\text{ENOB} = \log_2\left(1 + \left(\frac{V_{MAX} - V_{MIN}}{\Delta_{MAX}}\right)\right)$$

V_{MAX} and V_{MIN} are max and min outputs and Δ_{MAX} is maximum absolute step (HW problem)

Quantization noise:

$$\text{ENOB} = \frac{SNR_{dB}}{6.02}$$

rel to triangle/sawtooth

$$\text{ENOB} = \frac{SNR_{dB} - 1.76}{6.02}$$

rel to sinusoid

Stay Safe and Stay Healthy !

End of Lecture 29